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Coulomb wave function correction to Bose-Einstein correlations

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Abstract

We reexamine formulas for the Coulomb effects on the pion interferometry given by Pratt in 1986, and derive several correct formulas. Analytic expressions for this effect, i.e., the Coulomb correction to the Bose-Einstein correlations, are presented. Several numerical computations are shown.

Introduction: The study of the Bose-Einstein correlations (BEC) on pions and kaons is one of current problems. Recently the authors of NA44 experiment [1] have mentioned a paper on the Coulomb correction to the BEC [2]. However, since there is difference concerning treatments of the Gamow factor between Refs. [1, 2], we are interested in this subject and reexamine several formulas given in Ref. [2]. We have recognized that those formulas given by Pratt are very useful, and found that there are a few improper expressions in Eq. (3.13) of Ref. [2].

In the second paragraph, we recalculate several formulas in Ref. [2], and show numerical results. In the third paragraph, to consider the problem about the different treatments of the Gamow factor in Refs. [1, 2], we recalculate the BEC with the Coulomb correction and show numerical results. Concluding remarks are presented in the final paragraph.

Recalculation of formulas by Pratt: First of all we write down the Coulomb relative wave function [3]:

$$\Psi(\mathbf{q},\mathbf{r}) = \Gamma(1+i\gamma)e^{\pi\gamma/2}e^{i\mathbf{q}\cdot\mathbf{r}}\Phi(-i\gamma,1,iqr(1-\cos\theta)), \tag{1}$$

where $\gamma = m\alpha/2q$, and Φ is the confluent hypergeometric function:

$$\begin{aligned}\Phi(-i\gamma,1,iqr(1-\cos\theta)) &= 1+\sum_{n=1}^{\infty}(-i\gamma)(1-i\gamma)\cdots(n-1-i\gamma)\frac{(ix)^n}{(n!)^2}, \qquad (2)\\ \Gamma(1+i\gamma)\Gamma^*(1+i\gamma) &= \pi\gamma/\sinh(\pi\gamma),\end{aligned}$$

where $x = qr(1-\cos\theta)$. We use the following approximation for Φ, because of the usefulness [2]. The function Φ can be expanded in power of γ. To the first order in γ,

$$\begin{aligned}&\Phi(-i\gamma,1,ix) = 1+\gamma\,\mathrm{Si}(x) - i\gamma\,(\mathrm{Ci}(x)-C-\ln(x)), \qquad (3)\\ &\mathrm{Si}(x) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!(2n+1)},\\ &\mathrm{Ci}(x)-C-\ln(x) = \sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!(2n)},\end{aligned}$$

where $\mathrm{Si}(x)$ and $\mathrm{Ci}(x)$ are the sine and cosine integral, respectively. C is the Euler's number. After the angular integration in θ, we have the following expressions which correspond to Eq. (3.13) [2]. Notice that the second integrand is different from that of Ref. [2], and different arguments in $\mathrm{Si}(x)$, and $\mathrm{Ci}(x)-\ln(x)$ from Eq. (3.13) [2]:

$$\begin{aligned}I_1(q) &= \int_0^{\infty} 4\pi g(r)r^2 dr[1+2\gamma F(2qr)]\\ &= 1+\delta_{1C}, \qquad (4)\\ F(2qr) &= \sum_{n=0}^{\infty}\frac{(-1)^n(2qr)^{2n+1}}{(2n+1)!(2n+1)(2n+2)}.\\ I_2(q) &= \int_0^{\infty} 4\pi g(r)r^2 dr\left\{\frac{\sin(2qr)}{2qr}\right.\\ &\quad\left.+\gamma\left[\frac{\cos(2qr)}{2qr}(\mathrm{Ci}(2qr)-C-\ln(2qr))+\frac{\sin(2qr)}{2qr}\mathrm{Si}(2qr)\right]\right\}, \qquad (5)\end{aligned}$$

where $g(r)$ is the source function. Since in the present paper the Gaussian source function is assumed, we have to pay our attentions to mathematical property of the

Gaussian source functions:

$$
\begin{aligned}
\rho(r_1)\rho(r_2) &= \frac{1}{(2\pi)^{3/2}R^3}\exp\left(-\frac{r_1^2}{2R^2}\right)\frac{1}{(2\pi)^{3/2}R^3}\exp\left(-\frac{r_2^2}{2R^2}\right) \\
&= \frac{1}{(2\pi)^{3/2}(R/\sqrt{2})^3}\exp\left(-\frac{R_{cm}^2}{R^2}\right)\frac{1}{(2\pi)^{3/2}(\sqrt{2}R)^3}\exp\left(-\frac{r^2}{4R^2}\right) \\
&= g(R_{cm})g(r). \qquad (6)
\end{aligned}
$$

In the derivation of $I_2(q)$, we take the leading terms of the following integrations (with $A = 2qr$), according to [2]:

$$
\begin{aligned}
\Theta(2n+1) &= \int_{-1}^{1}(1-\cos\theta)^{2n+1}\cos(A\cos\theta)d\cos\theta, \qquad (7)\\
\Theta(1) &= \frac{2\sin A}{A},\\
\Theta(3) &= \frac{2^3\sin A}{A} + \frac{12\cos A}{A^2} - \frac{12\sin A}{A^3},\\
&\cdots\cdots,\\
\Theta(2n+1)^{leading} &= \frac{2^{2n+1}\sin A}{A},\\
\Theta(2n) &= \mp\int_{-1}^{1}(1\pm\cos\theta)^{2n}\sin(A\cos\theta)d\cos\theta, \qquad (8)\\
\Theta(2) &= \frac{2^2\cos A}{A} - \frac{2^2\sin A}{A^2},\\
\Theta(4) &= \frac{2^4\cos A}{A} - \frac{2^5\sin A}{A^2} - \frac{3\cdot 2^4\cos A}{A^3} + \frac{3\cdot 2^4\cos A}{A^4},\\
&\cdots\cdots,\\
\Theta(2n)^{leading} &= \frac{2^{2n}\cos A}{A}.
\end{aligned}
$$

Here we show numerical results a la Pratt: The author of Ref. [2] has used the following formula with the squared Gamow factor,

$$
R_{CC}^{Pratt} = G(q)^2[I_1(q)^2 + I_2(q)^2], \qquad (9)
$$

where $G(q) = 2\pi\gamma/(e^{2\pi\gamma}-1)$. To confirm FIG. 2 in Ref. [2], we use Eq. (3.13) with the above formula, $C^{Pratt}(k=2q) = R_{CC}^{Pratt}/G(q)^2$. (Notice that FIG. 2 (a) and (c) in Ref. [2] are reversed.) Indeed, our numerical computations reproduce FIG. 2 in Ref. [2]. For the sake of comparisons, our new results in terms of Eqs. (4), (5),

(6) and (9) with the same parameters are shown in FIG. 2. Due to correct factor $1/(2n+2)$ in Eq. (4), the intercepts at $k = 0$ (MeV/c) are smaller than those of FIG. 1.

Reformulation of Eq. (9): As explained in a previous paragraph, the author of Ref. [2] has used the squared Gamow factor. However, the authors of Ref. [1] have used the single Gamow factor [4]. By making use of a paper by Bowler [5], we obtain the single Gamow factor. In other words, the single Gamow factor seems to be reasonable [6, 7], because of property of the Coulomb wave function for the system of the two charged identical bosons [3]. Actually by making use of Eq. (1) and the following symmetrized Coulomb wave function,

$$\Psi_S(\mathbf{q}, \mathbf{r}) = \Gamma(1 + i\gamma)e^{\pi\gamma/2}e^{-i\mathbf{q}\cdot\mathbf{r}}\Phi(-i\gamma, 1, iqr(1 + \cos\theta)), \tag{10}$$

we have the following correction formula for the exchange function, after the angular integration in θ:

$$\begin{aligned} I_2(q) &= 4\pi \int_0^\infty g(r)r^2dr \left\{ \frac{\sin A}{A} + \gamma(\mathrm{Sp}(qr) + \mathrm{Cp}(qr)) \right\}, \\ &= E_{2B} + \delta_{EC}, \end{aligned} \tag{11}$$

where

$$\begin{aligned} \mathrm{Sp}(qr) &= \sum_{n=0}^{\infty} \frac{(-1)^{2n+1}(qr)^{2n+1}\Theta(2n+1)}{(2n+1)!(2n+1)}, \\ \mathrm{Cp}(qr) &= \sum_{n=1}^{\infty} \frac{(-1)^{2n}(qr)^{2n}\Theta(2n)}{(2n)!(2n)}. \end{aligned}$$

In conclusion, we have the following formula:

$$R_{CC} = G(q)[1 + \delta_{1C} + E_{2B} + \delta_{EC}]. \tag{12}$$

To compare data corrected by only the Gamow factor with theoretical calculations, we should consider the ratio as

$$\begin{aligned} C(k = 2q) &= R_{CC}/G(q) \\ &= (1 + \delta_{1C} + \delta_{EC}) \left[1 + \frac{E_{2B}}{1 + \delta_{1C} + \delta_{EC}} \right]. \end{aligned} \tag{13}$$

It should be notice that the normalization and an effective degree of coherence, i.e., the denominator of the ratio $E_{2B}/(1+\delta_{1C}+\delta_{EC})$ is relating to each other. We show our results of the BEC with Coulomb corrections in FIG. 3. As is seen in FIG.3 (d) and (e), it should be noticed that the numerical results depend on the source size (R), through $\Theta(n)$. In other words, there are discrepancies between contributions of the leading terms $\Theta(n)^{leading}$ and exact expressions of $\Theta(n)$, as the source size becomes large.

To apply the above equation to data corrected by the Coulomb wave function [8], we should modify the formula as:

$$\begin{aligned} C(k=2q)^{[CC]} &= \frac{R_{CC}}{[G(q)(1+\delta_{1C}+\delta_{EC})]} \\ &= 1+\frac{E_{2B}}{1+\delta_{1C}+\delta_{EC}}. \end{aligned} \tag{14}$$

The denominator of the ratio $E_{2B}/(1+\delta_{1C}+\delta_{EC})$ is also playing the effective degree of coherence.

Concluding remarks: We reexamine several formulas in Ref. [2]. Moreover, we obtain several improved formulas for Coulomb correction to the BEC, by the use of the approximation of the first order of γ. These formulas can be used in analyses of the BEC in which the final state interactions excluding the Coulomb effects are weak.

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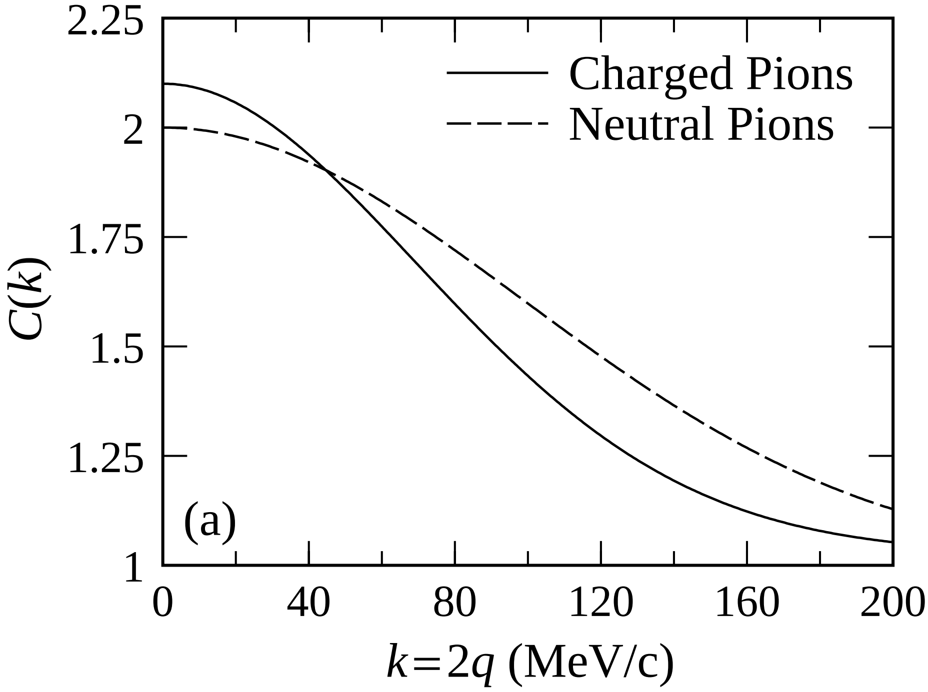
Figure Captions

Fig. 1. To reproduce FIG. 2 of Ref. [2], we use the same parameters and Eq. (3.13) of Ref. [2] with $g(r) = \frac{1}{(2\pi)^{3/2}R^3}\exp(-\frac{r^2}{2R^2})$. $R = 2$, 4, 8 fm are used in (a),(b), and (c), respectively.The dotted lines are obtained by $g(r) = \frac{1}{(\pi)^{3/2}R^3}\exp(-\frac{r^2}{R^2})$. They are coincided with FIG.2 of Ref. [2].

Fig. 2. Numerical computations in terms of Eqs. (4), (5) and (9). We use the correct expressions with $g(r) = \frac{1}{(2\pi)^{3/2}R^3}\exp(-\frac{r^2}{2R^2})$ in the framework of Pratt. $R = 2$, 4, 8 fm are used in (a),(b), and (c), respectively.

Fig. 3. Numerical computations in terms of Eqs. (4), (5), (6), (11), and (12). Here we use $g(r) = \frac{1}{(2\pi)^{3/2}(\sqrt{2}R)^3}\exp(-\frac{r^2}{4R^2})$, because of Eq. (6). The solid lines are obtained by the exact expression of $\Theta(n)$. The dotted lines are obtained by making use of $\Theta(n)^{leading}$. $R = 2$, 4, 8, 12, and 16 fm are used in (a),(b), (c), (d), and (e), respectively.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Charged Pions
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(a)

Fig. 1

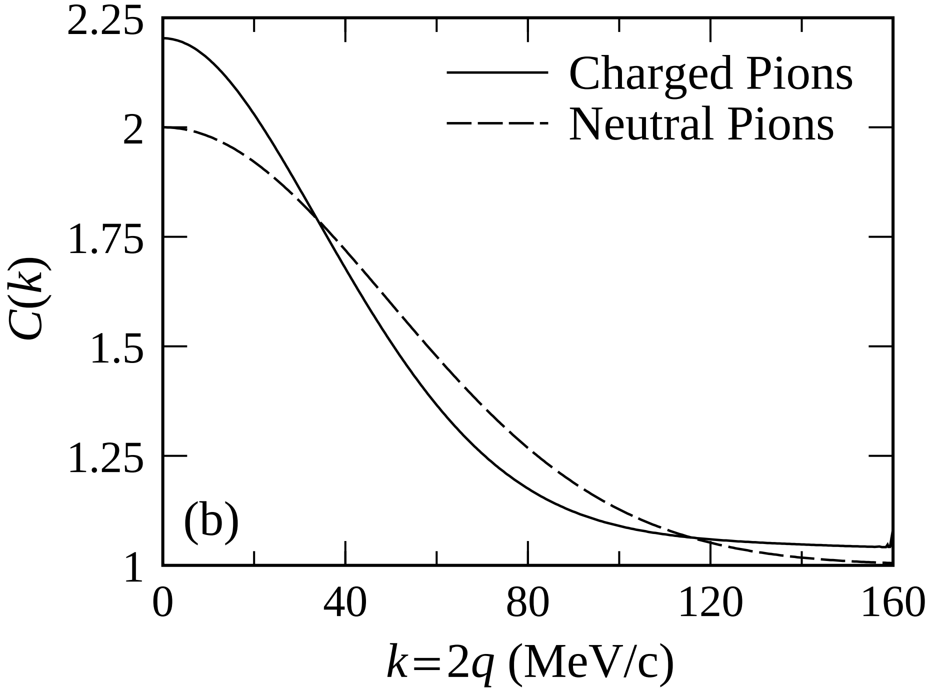
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Fig. 1

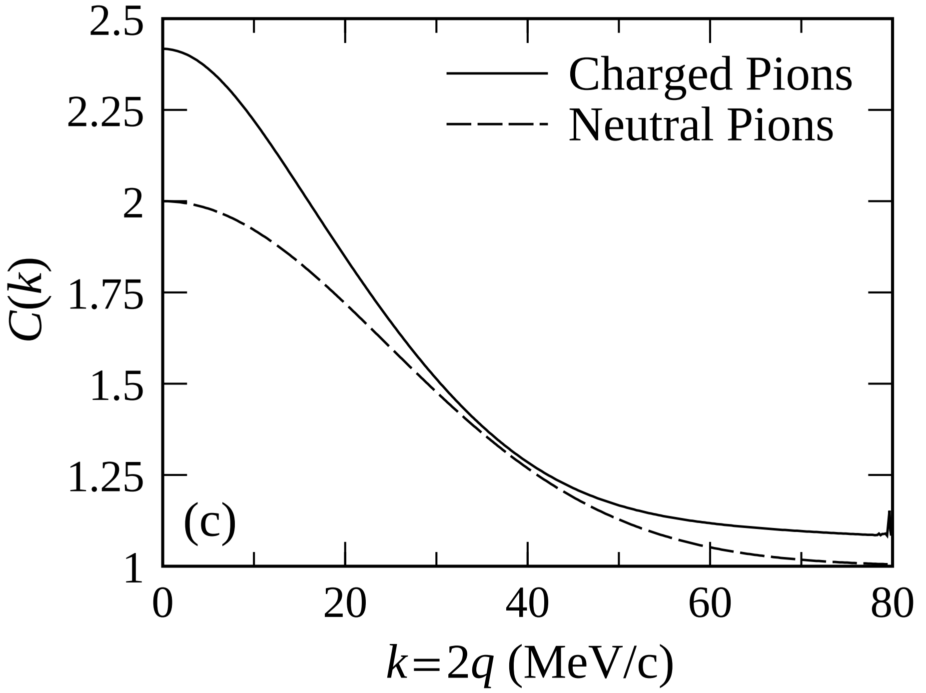
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Fig. 1

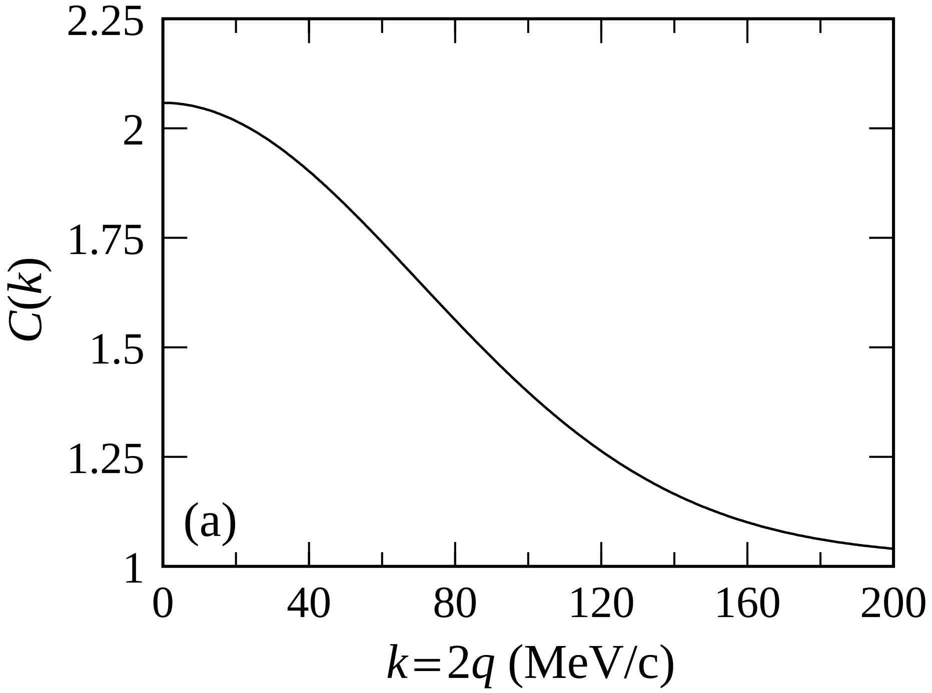
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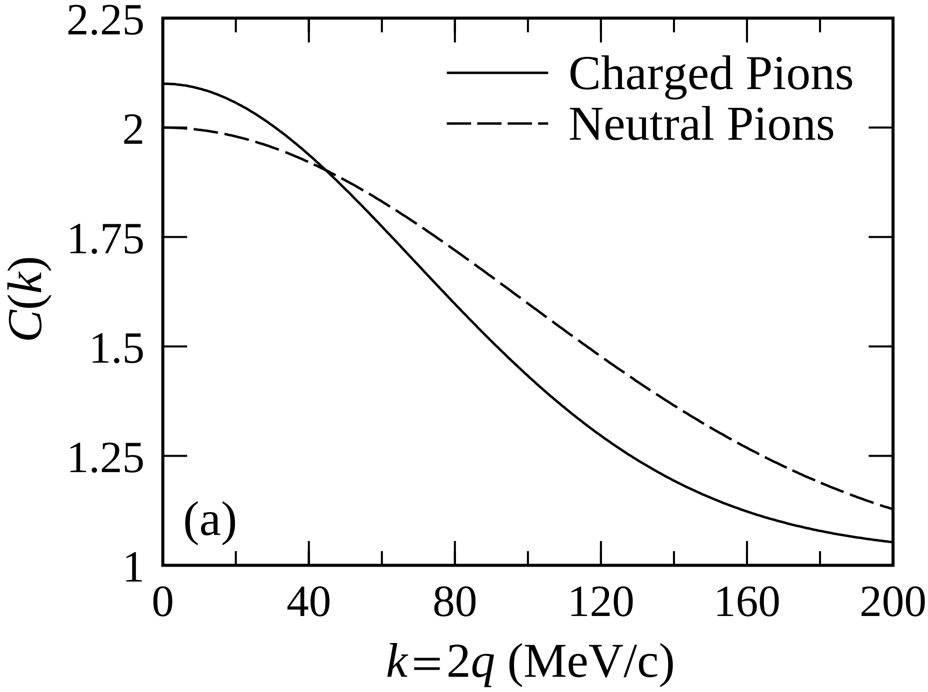
Fig. 2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Neutral Pions
(a)

Fig. 1

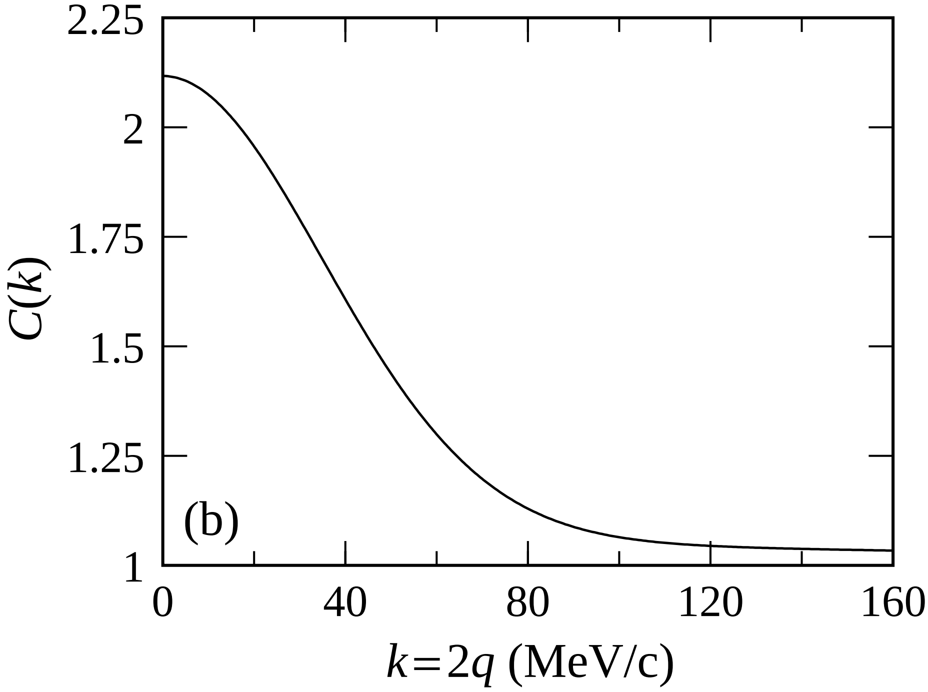
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Fig. 2

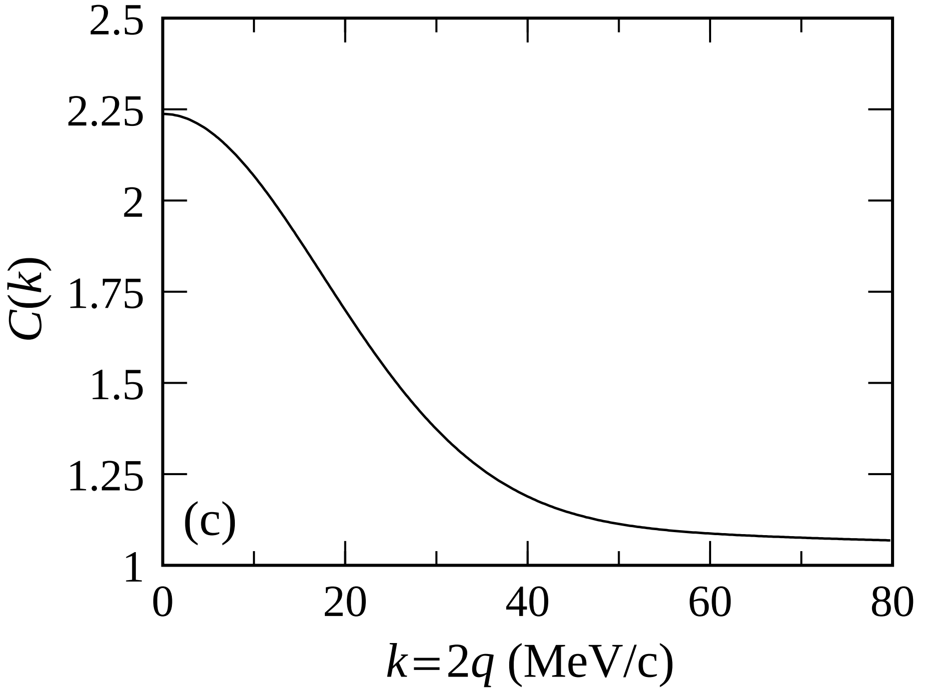
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Fig. 2

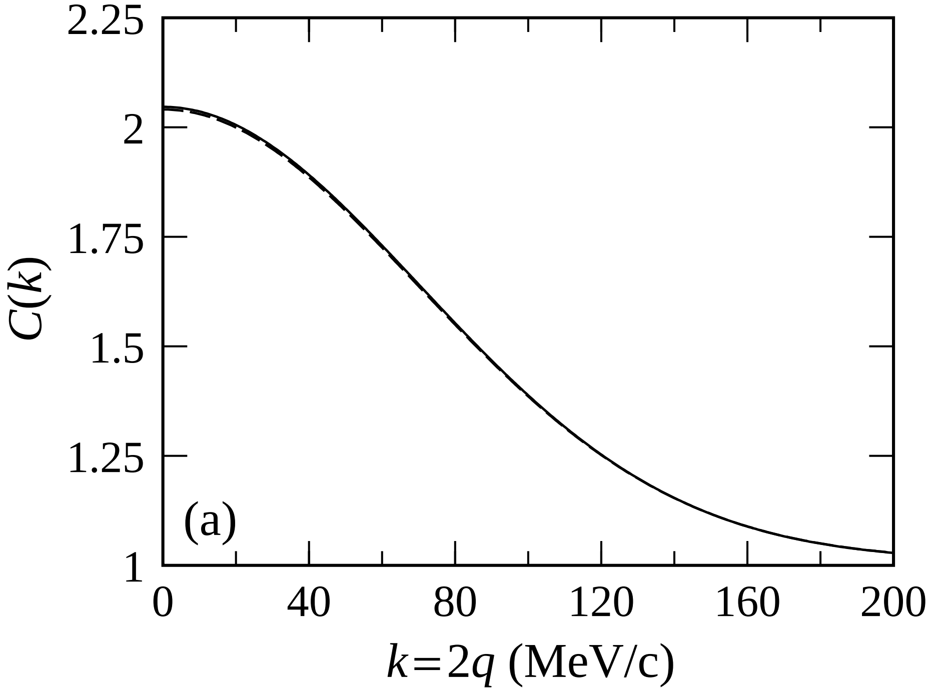
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Fig. 3

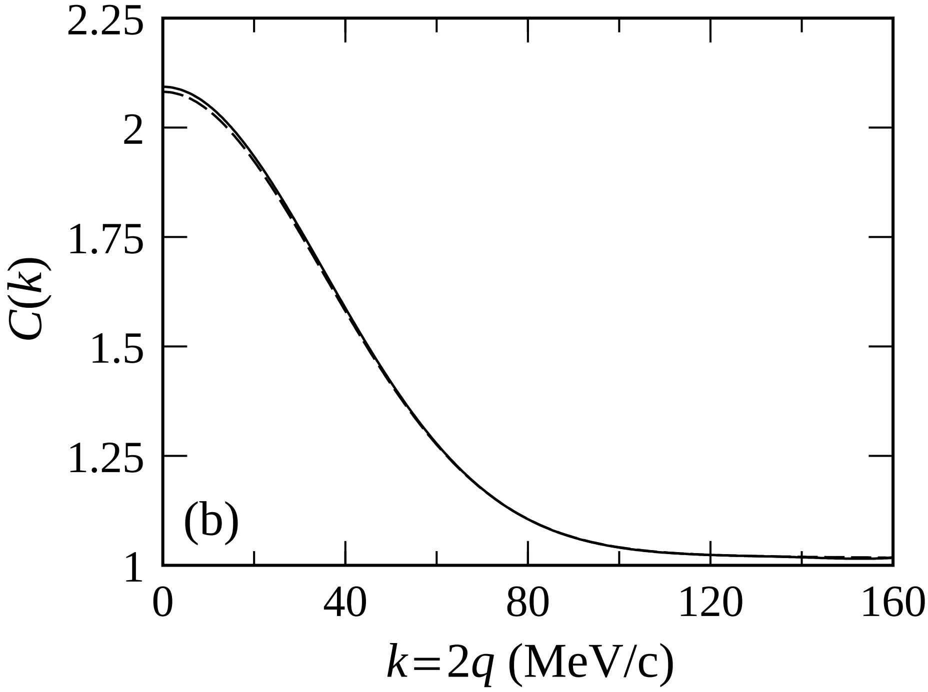
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Fig. 3

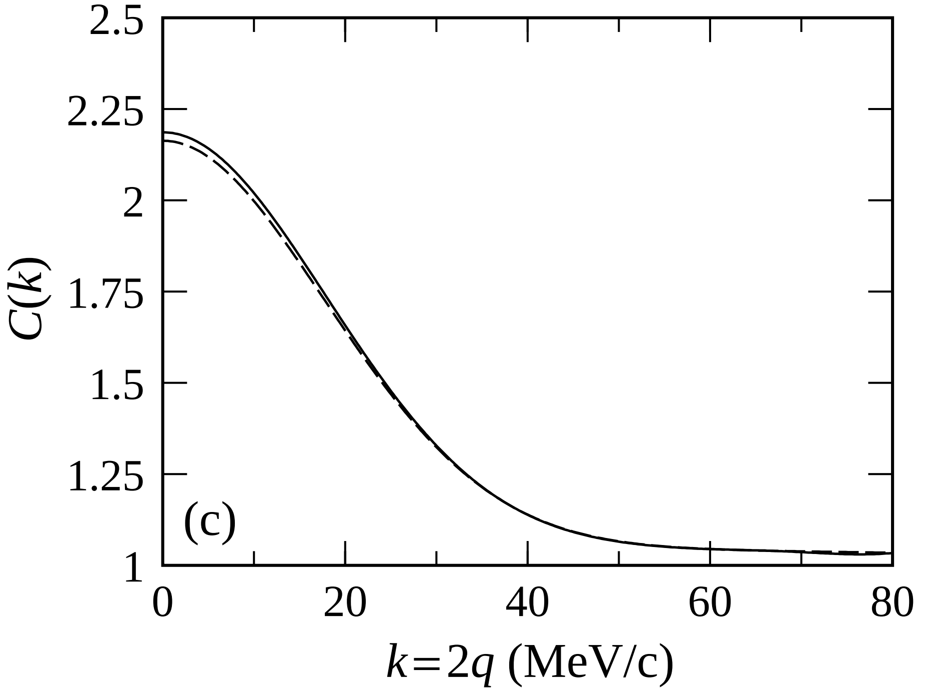
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Fig. 3

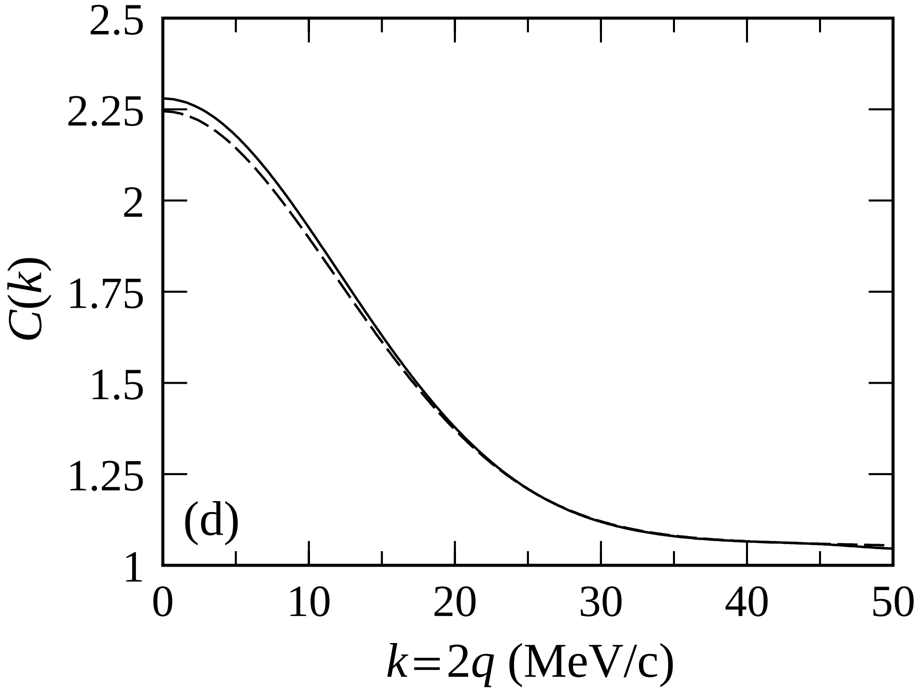
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Fig. 3

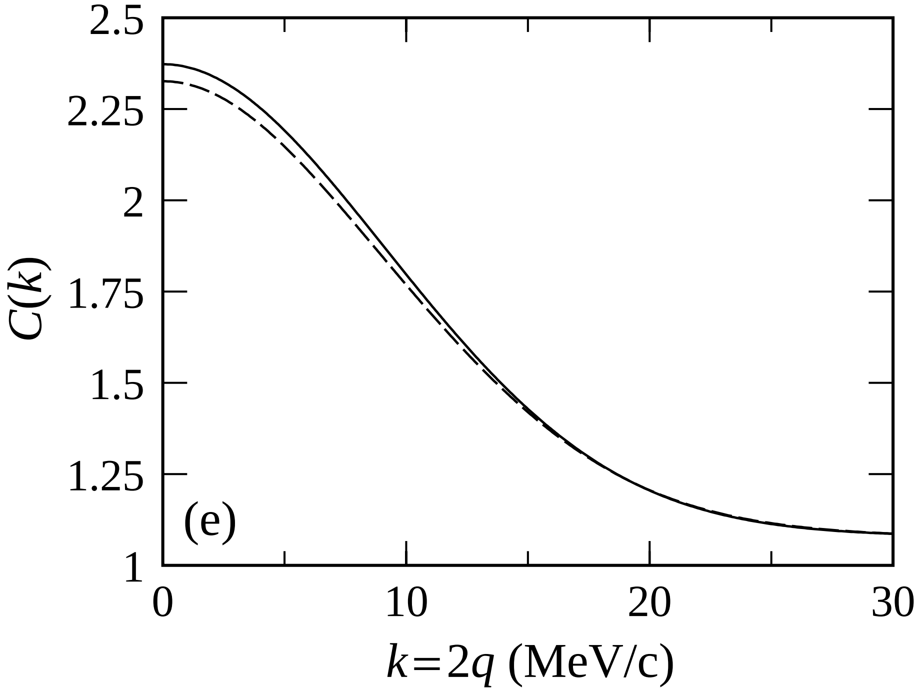
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Fig. 3