物理数学 I の宿題 [ver.2.1.1] の答 (一部)

ミスプリを見つけたら連絡して下さい。

§1 の宿題

1.

2.

3.

4. $\begin{pmatrix} -240 \\ 282 \\ 108 \end{pmatrix}, \begin{pmatrix} -58 \\ 118 \\ -150 \end{pmatrix}, \begin{pmatrix} -240 \\ 282 \\ 108 \end{pmatrix} + \begin{pmatrix} 182 \\ -164 \\ -258 \end{pmatrix} + \begin{pmatrix} 58 \\ -118 \\ 150 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

5. $\vec{0}$

6.

§2 の宿題

1. $\dot{k}\vec{a} + k\dot{\vec{a}}, \quad \dot{\vec{a}} \cdot \vec{b} + \vec{a} \cdot \dot{\vec{b}}, \quad \dot{\vec{a}} \times \vec{b} + \vec{a} \times \dot{\vec{b}}$

2.

3.

4. $\begin{pmatrix} 3t^2 - 4 \\ 2t + 4 \\ 16t - 9t^2 \end{pmatrix}, \begin{pmatrix} 6t \\ 2 \\ 16 - 18t \end{pmatrix}, \frac{16}{3}\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \frac{2}{3}\begin{pmatrix} 2 \\ -13 \\ -22 \end{pmatrix}$

5. $\dot{\vec{r}} = \dot{r}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} + r\dot{\theta}\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}, \quad \ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$

6. $\begin{pmatrix} 6xy \\ 3x^2 - 3y^2z^2 \\ -2y^3z \end{pmatrix}, 2xz - 6y^2z^2 + xy^2, \begin{pmatrix} 2xyz + 4y^3z \\ x^2 - y^2z \\ 0 \end{pmatrix}, 6y - 6yz^2 - 2y^3, \begin{pmatrix} 2z \\ -12yz^2 - 4y^3 \\ 2xz \end{pmatrix}$

7. $\begin{pmatrix} 2xyz \\ x^2z \\ x^2y \end{pmatrix}, 2z^2 - z + 9xz^2, \begin{pmatrix} y \\ 4xz - 3z^3 \\ 0 \end{pmatrix}, 2yz, \begin{pmatrix} 4x \\ 0 \\ 18xz \end{pmatrix}$

8. $\frac{f'(r)}{r}\vec{r}, \; f''(r) + \frac{2}{r}f'(r)$

9. $rf'(r) + 3f(r), \; \vec{0}, \; f''(r)\vec{r} + 4\frac{f'(r)}{r}\vec{r}$

10. $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\frac{\vec{r}}{r}$

11. $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\Big(3\dfrac{\vec{d}\cdot\vec{r}}{r^5}\vec{r} - \dfrac{1}{r^3}\vec{d}\Big)$

12. $\vec{B} = \dfrac{\mu_0}{4\pi}\Big(3\dfrac{\vec{m}\cdot\vec{r}}{r^5}\vec{r} - \dfrac{1}{r^3}\vec{m}\Big)$

13. $\pm\frac{1}{3}\begin{pmatrix}-1\\2\\2\end{pmatrix}$

14. $2\vec{\omega}$

15. (1) $\vec{\nabla}\times(\vec{\nabla}\phi) = \vec{0}$, $\mathrm{rot}\,\mathrm{grad}\,\phi = \vec{0}$
(2) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{a}) = 0$, $\mathrm{div}\,\mathrm{rot}\,\vec{a} = 0$
(3) $\vec{\nabla}\times(\vec{\nabla}\times\vec{a}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{a}) - \triangle\,\vec{a}$, $\mathrm{rot}\,\mathrm{rot}\,\vec{a} = \mathrm{grad}\,\mathrm{div}\,\vec{a} - \triangle\,\vec{a}$
(4) $\vec{\nabla}(\phi\psi) = \psi\vec{\nabla}\phi + \phi\vec{\nabla}\psi$, $\mathrm{grad}\,(\phi\psi) = \psi\,\mathrm{grad}\,\phi + \phi\,\mathrm{grad}\,\psi$
(5) $\vec{\nabla}\cdot(\phi\vec{a}) = \vec{a}\cdot\vec{\nabla}\phi + \phi\vec{\nabla}\cdot\vec{a}$, $\mathrm{div}\,(\phi\vec{a}) = \vec{a}\cdot\mathrm{grad}\,\phi + \phi\,\mathrm{div}\,\vec{a}$
(6) $\vec{\nabla}\times(\phi\vec{a}) = (\vec{\nabla}\phi)\times\vec{a} + \phi\vec{\nabla}\times\vec{a}$, $\mathrm{rot}\,(\phi\vec{a}) = \mathrm{grad}\,\phi\times\vec{a} + \phi\,\mathrm{rot}\,\vec{a}$
(7) $\vec{\nabla}(\vec{a}\cdot\vec{b}) = (\vec{a}\cdot\vec{\nabla})\vec{b} + (\vec{b}\cdot\vec{\nabla})\vec{a} + \vec{a}\times(\vec{\nabla}\times\vec{b}) + \vec{b}\times(\vec{\nabla}\times\vec{a})$,
$\mathrm{grad}\,(\vec{a}\cdot\vec{b}) = (\vec{a}\cdot\mathrm{grad}\,)\vec{b} + (\vec{b}\cdot\mathrm{grad}\,)\vec{a} + \vec{a}\times\mathrm{rot}\,\vec{b} + \vec{b}\times\mathrm{rot}\,\vec{a}$
(8) $\vec{\nabla}\cdot(\vec{a}\times\vec{b}) = (\vec{\nabla}\times\vec{a})\cdot\vec{b} - \vec{a}\cdot(\vec{\nabla}\times\vec{b})$, $\mathrm{div}\,(\vec{a}\times\vec{b}) = \vec{b}\cdot\mathrm{rot}\,\vec{a} - \vec{a}\cdot\mathrm{rot}\,\vec{b}$
(9) $\vec{\nabla}\times(\vec{a}\times\vec{b}) = \vec{a}(\vec{\nabla}\cdot\vec{b}) - \vec{b}(\vec{\nabla}\cdot\vec{a}) + (\vec{b}\cdot\vec{\nabla})\vec{a} - (\vec{a}\cdot\vec{\nabla})\vec{b}$,
$\mathrm{rot}\,(\vec{a}\times\vec{b}) = \vec{a}\,\mathrm{div}\,\vec{b} - \vec{b}\,\mathrm{div}\,\vec{a} + (\vec{b}\cdot\mathrm{grad}\,)\vec{a} - (\vec{a}\cdot\mathrm{grad}\,)\vec{b}$
(10) $\triangle\,(\phi\psi) = \psi\triangle\,\phi + \phi\triangle\,\psi + 2\vec{\nabla}\phi\cdot\vec{\nabla}\psi$, $\triangle\,(\phi\psi) = \psi\triangle\,\phi + \phi\triangle\,\psi + 2\mathrm{grad}\,\phi\cdot\mathrm{grad}\,\psi$
証明は略

16. $\vec{k}\cdot\vec{E}_0 = 0$, $\vec{k}\cdot\vec{B}_0 = 0$, $\vec{k}\times\vec{E}_0 = \omega\vec{B}_0$, $\vec{k}\times\vec{B}_0 = -\frac{1}{c^2}\omega\vec{E}_0$. $(\Rightarrow \vec{k}^{\,2} = \frac{\omega^2}{c^2})$

§3の宿題

法線の向きの指定のない問題では，どちらか一つの向きを選んで計算した値である。

1. (1) $\vec{v}_0 + \begin{pmatrix}-\rho\omega\sin\omega t\\ \rho\omega\cos\omega t - \rho\omega\\ 0\end{pmatrix}$ (2) $\vec{x}_0 + \vec{v}_0 t + \begin{pmatrix}\rho\cos\omega t - \rho\\ \rho\sin\omega t - \rho\omega t\\ 0\end{pmatrix}$

2. $\frac{1}{4}M(a^2+b^2)$

3. $\dfrac{2}{5}M\dfrac{a^5-b^5}{a^3-b^3}$

4. (i) $\frac{31}{6}\sqrt{3}$ (ii) $1+\frac{13}{2}\sqrt{2}$ (iii) $\frac{1}{6}(14\sqrt{14}-1)$ (iv) 0

5. (i) $\frac{7}{2}$ (ii) $\frac{15}{2}$ (iii) $\frac{64}{15}$ (iv) 0

6. (i) 2 (ii) 2 (iii) 2 (iv) 0

7. (1) $\mathrm{rot}\,\vec{f} = \vec{0}$ (2) $-x^2y - xz^3 + \mathrm{const.}$ (3) 2

8. $\begin{pmatrix} \frac{8}{11} \\ \frac{4}{5} \\ 1 \end{pmatrix}$

9. $\begin{pmatrix} -\frac{9}{10} \\ -\frac{2}{3} \\ \frac{7}{5} \end{pmatrix}$

10. $\vec{B} = \dfrac{\mu_0 I}{4\pi} \dfrac{2\pi a^2}{(a^2+z^2)^{\frac{3}{2}}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

11. $\vec{B} = \dfrac{\mu_0 I}{2\pi(x^2+y^2)} \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} = \dfrac{\mu_0 I}{2\pi\sqrt{x^2+y^2}} \begin{pmatrix} \frac{-y}{\sqrt{x^2+y^2}} \\ \frac{x}{\sqrt{x^2+y^2}} \\ 0 \end{pmatrix}$

12. 24

13. 参考：$120 = -48 + 60 + 108$

14. $\begin{pmatrix} 100 \\ 100 \\ 0 \end{pmatrix}$

15. 90

16. 参考：$-580 = 4\pi - 600 + (-4\pi + 20) + 0$

17. 参考：π

18. 参考：$2 = 0 + 0 + 0 + 2 - \frac{1}{2} + \frac{1}{2}$

19. $\frac{2}{15}$

20. 参考：$\frac{8}{3} = 0 - 1 + 0 + \frac{11}{3}$

21. 参考：$84\pi = 36\pi + 0 + 48\pi$

§4の問題

1.

2. (1) $\vec{e}_r = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix},\ \vec{e}_\theta = \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix},\ \vec{e}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$

   $\dot{\vec{r}} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta + \dot{z}\vec{e}_z,\ \ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{e}_\theta + \ddot{z}\vec{e}_z$

   (2) $\vec{e}_r = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix},\ \vec{e}_\theta = \begin{pmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ -\sin\theta \end{pmatrix},\ \vec{e}_\varphi = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix},\ \dot{\vec{r}} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta + r\sin\theta\,\dot{\varphi}\vec{e}_\varphi,$

   $\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\,\dot{\varphi}^2)\vec{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta} - r\sin\theta\cos\theta\,\dot{\varphi}^2)\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{d}{dt}(r^2\sin^2\theta\,\dot{\varphi})\vec{e}_\varphi$

3. (1) $\mathrm{grad}\,\phi=\frac{\partial\phi}{\partial r}\vec{e}_r+\frac{1}{r}\frac{\partial\phi}{\partial\theta}\vec{e}_\theta+\frac{\partial\phi}{\partial z}\vec{e}_z,\ \ \mathrm{div}\,\vec{a}=\frac{1}{r}\frac{\partial}{\partial r}(ra_r)+\frac{1}{r}\frac{\partial a_\theta}{\partial\theta}+\frac{\partial a_z}{\partial z},$

$\mathrm{rot}\,\vec{a}=\Big(\frac{1}{r}\frac{\partial a_z}{\partial\theta}-\frac{\partial a_\theta}{\partial z}\Big)\vec{e}_r+\Big(\frac{\partial a_r}{\partial z}-\frac{\partial a_z}{\partial r}\Big)\vec{e}_\theta+\Big(\frac{1}{r}\frac{\partial}{\partial r}(ra_\theta)-\frac{1}{r}\frac{\partial a_r}{\partial\theta}\Big)\vec{e}_z,$

$\triangle\,\phi=\frac{1}{r}\frac{\partial}{\partial r}\big(r\frac{\partial\phi}{\partial r}\big)+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}+\frac{\partial^2\phi}{\partial z^2}$

(2) $\mathrm{grad}\,\phi=\frac{\partial\phi}{\partial r}\vec{e}_r+\frac{1}{r}\frac{\partial\phi}{\partial\theta}\vec{e}_\theta+\frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}\vec{e}_\varphi,$

$\mathrm{div}\,\vec{a}=\frac{1}{r^2\sin\theta}\Big(\sin\theta\frac{\partial}{\partial r}(r^2a_r)+r\frac{\partial}{\partial\theta}(\sin\theta\,a_\theta)+r\frac{\partial a_\varphi}{\partial\varphi}\Big),$

$\mathrm{rot}\,\vec{a}=\frac{1}{r^2\sin\theta}\Big(\frac{\partial}{\partial\theta}(r\sin\theta\,a_\varphi)-\frac{\partial}{\partial\varphi}(ra_\theta)\Big)\vec{e}_r+\frac{1}{r\sin\theta}\Big(\frac{\partial a_r}{\partial\varphi}-\frac{\partial}{\partial r}(r\sin\theta\,a_\varphi)\Big)\vec{e}_\theta$

$+\frac{1}{r}\Big(\frac{\partial}{\partial r}(ra_\theta)-\frac{\partial a_r}{\partial\theta}\Big)\vec{e}_\varphi,$

$\triangle\,\phi=\frac{1}{r^2\sin\theta}\Big(\sin\theta\frac{\partial}{\partial r}\big(r^2\frac{\partial\phi}{\partial r}\big)+\frac{\partial}{\partial\theta}\big(\sin\theta\frac{\partial\phi}{\partial\theta}\big)+\frac{1}{\sin\theta}\frac{\partial^2\phi}{\partial\varphi^2}\Big)$

4. (4) $e=\sqrt{1+\frac{2}{m}\Big(\frac{L}{A}\Big)^2E}$

(ついでに) 線型代数の問題

問 1,2,3 の行列を A とおく。P の取り方は一意的ではない。

1. (1) $P=\begin{pmatrix}2&1\\1&1\end{pmatrix},\quad P^{-1}AP=\begin{pmatrix}2&0\\0&3\end{pmatrix},\quad A=2\cdot\begin{pmatrix}2&-2\\1&-1\end{pmatrix}+3\cdot\begin{pmatrix}-1&2\\-1&2\end{pmatrix}$

(2) $P=\begin{pmatrix}1&1&1\\1&0&1\\1&1&0\end{pmatrix},\quad P^{-1}AP=\begin{pmatrix}1&0&0\\0&2&0\\0&0&3\end{pmatrix},$

$A=1\cdot\begin{pmatrix}-1&1&1\\-1&1&1\\-1&1&1\end{pmatrix}+2\cdot\begin{pmatrix}1&-1&0\\0&0&0\\1&-1&0\end{pmatrix}+3\cdot\begin{pmatrix}1&0&-1\\1&0&-1\\0&0&0\end{pmatrix}$

(3) $P=\begin{pmatrix}-2&-2&-3\\-1&1&0\\1&0&1\end{pmatrix},\quad P^{-1}AP=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix},$

$A=1\cdot\begin{pmatrix}2&4&6\\1&2&3\\-1&-2&-3\end{pmatrix}-1\cdot\begin{pmatrix}-1&-4&-6\\-1&-1&-3\\1&2&4\end{pmatrix}$

(4) $P=\begin{pmatrix}1&1&0&1\\0&-2&1&-1\\-1&0&-1&0\\1&1&0&2\end{pmatrix},\quad P^{-1}AP=\begin{pmatrix}0&0&0&0\\0&2&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix},$

$$A = 0 \cdot \begin{pmatrix} 3 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ -3 & -1 & -1 & 1 \\ 3 & 1 & 1 & -1 \end{pmatrix} + 2 \cdot \begin{pmatrix} -1 & -1 & -1 & 0 \\ 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} -1 & 0 & 0 & 1 \\ -2 & -1 & -2 & 0 \\ 3 & 1 & 2 & -1 \\ -2 & 0 & 0 & 2 \end{pmatrix}$$

2. (1) $P = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 9 & 0 \\ 0 & -1 \end{pmatrix}, \quad A = 9 \cdot \frac{1}{5} \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} - 1 \cdot \frac{1}{5} \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$

(2) $P = \frac{1}{3} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix},$

$$A = -1 \cdot \frac{1}{9} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \\ 2 & 4 & 4 \end{pmatrix} + 2 \cdot \frac{1}{9} \begin{pmatrix} 4 & 2 & -4 \\ 2 & 1 & -2 \\ -4 & -2 & 4 \end{pmatrix} + 5 \cdot \frac{1}{9} \begin{pmatrix} 4 & -4 & 2 \\ -4 & 4 & -2 \\ 2 & -2 & 1 \end{pmatrix}$$

(3) $P = \begin{pmatrix} \frac{2}{3} & \frac{1}{\sqrt{5}} & \frac{4}{3\sqrt{5}} \\ \frac{-2}{3} & 0 & \frac{5}{3\sqrt{5}} \\ \frac{1}{3} & \frac{-2}{\sqrt{5}} & \frac{2}{3\sqrt{5}} \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 8 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$

$$A = 8 \cdot \frac{1}{9} \begin{pmatrix} 4 & -4 & 2 \\ -4 & 4 & -2 \\ 2 & -2 & 1 \end{pmatrix} - 1 \cdot \frac{1}{9} \begin{pmatrix} 5 & 4 & -2 \\ 4 & 5 & 2 \\ -2 & 2 & 8 \end{pmatrix}$$

(4) $P = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & 1 \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix},$

$$A = 3 \cdot \frac{1}{3} \begin{pmatrix} 2 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix} - 3 \cdot \frac{1}{3} \begin{pmatrix} 1 & 1 & -1 & 0 \\ 1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 1 \end{pmatrix}$$

3. (1) $P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}, \quad A = 0 \cdot \frac{1}{2} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix} + 2 \cdot \frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$

(2) $P = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2i}{\sqrt{6}} & \frac{i}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{pmatrix},$

$$A = 2 \cdot \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} + 6 \cdot \frac{1}{6} \begin{pmatrix} 1 & 2i & -1 \\ -2i & 4 & 2i \\ -1 & -2i & 1 \end{pmatrix} + 3 \cdot \frac{1}{3} \begin{pmatrix} 1 & -i & -1 \\ i & 1 & -i \\ -1 & i & 1 \end{pmatrix}$$

(3) $P = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2i}{\sqrt{6}} & \frac{i}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$

$$A = 1 \cdot \frac{1}{3}\begin{pmatrix} 2 & i & 1 \\ -i & 2 & i \\ 1 & -i & 2 \end{pmatrix} - 2 \cdot \frac{1}{3}\begin{pmatrix} 1 & -i & -1 \\ i & 1 & -i \\ -1 & i & 1 \end{pmatrix}$$

(4) $P = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 0 & 1 \\ i & 0 & \sqrt{2} & -i \\ -1 & \sqrt{2} & 0 & -1 \\ -i & 0 & \sqrt{2} & i \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix},$

$$A = -1 \cdot \frac{1}{4}\begin{pmatrix} 1 & -i & -1 & i \\ i & 1 & -i & -1 \\ -1 & i & 1 & -i \\ -i & -1 & i & 1 \end{pmatrix} + 3 \cdot \frac{1}{4}\begin{pmatrix} 3 & i & 1 & -i \\ -i & 3 & i & 1 \\ 1 & -i & 3 & i \\ i & 1 & -i & 3 \end{pmatrix}$$

4. (1) $\frac{1}{3}\left(\begin{smallmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{smallmatrix}\right)\vec{x}$ (2) $\vec{x}'$

5. (1) $\frac{1}{5}\left(\begin{smallmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{smallmatrix}\right)\vec{x}$ (2) $\vec{x}'$ (3) 共に $\frac{1}{5}(x_2 + 2x_3)(y_2 + 2y_3)$

6. (1) 2 (2) $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, 4 (4) $\frac{1}{3}\begin{pmatrix} 4 & -1 & 1 \\ -1 & 4 & -1 \\ 1 & -1 & 4 \end{pmatrix}$

7. (1) $\frac{e^2}{2}\begin{pmatrix} e + e^{-1} & -(e - e^{-1}) \\ -(e - e^{-1}) & e + e^{-1} \end{pmatrix}$ (2) $e^{\alpha}\begin{pmatrix} 1 & 1 & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

(3) $\frac{1}{6}\begin{pmatrix} e + 3e^3 + 2e^{-2} & -2e + 2e^{-2} & 3e + 3e^3 - 6e^{-2} \\ -4e + 6e^3 - 2e^{-2} & 8e - 2e^{-2} & -12e + 6e^3 + 6e^{-2} \\ -2e + 3e^3 - 2e^{-2} & 4e - 2e^{-2} & -6e + 3e^3 + 6e^{-2} \end{pmatrix}$

8. (4) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$