

物理数学 I の演習問題（小竹）の答え（一部）

ミスプリに気付いたら連絡して下さい。

宿題 §1

問 1,2,3 の行列を A とおく。 P の取り方は一意的ではない。

$$1. (1) P = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}, \quad A = 2 \cdot \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} + 3 \cdot \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$$

$$(2) P = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix},$$

$$A = 1 \cdot \begin{pmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix} + 2 \cdot \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{pmatrix} + 3 \cdot \begin{pmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(3) P = \begin{pmatrix} -2 & -2 & -3 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

$$A = 1 \cdot \begin{pmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix} - 1 \cdot \begin{pmatrix} -1 & -4 & -6 \\ -1 & -1 & -3 \\ 1 & 2 & 4 \end{pmatrix}$$

$$(4) P = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & -1 \\ -1 & 0 & -1 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$A = 0 \cdot \begin{pmatrix} 3 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ -3 & -1 & -1 & 1 \\ 3 & 1 & 1 & -1 \end{pmatrix} + 2 \cdot \begin{pmatrix} -1 & -1 & -1 & 0 \\ 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} -1 & 0 & 0 & 1 \\ -2 & -1 & -2 & 0 \\ 3 & 1 & 2 & -1 \\ -2 & 0 & 0 & 2 \end{pmatrix}$$

$$2. (1) P = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 9 & 0 \\ 0 & -1 \end{pmatrix}, \quad A = 9 \cdot \frac{1}{5} \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} - 1 \cdot \frac{1}{5} \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$$

$$(2) P = \frac{1}{3} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix},$$

$$A = -1 \cdot \frac{1}{9} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \\ 2 & 4 & 4 \end{pmatrix} + 2 \cdot \frac{1}{9} \begin{pmatrix} 4 & 2 & -4 \\ 2 & 1 & -2 \\ -4 & -2 & 4 \end{pmatrix} + 5 \cdot \frac{1}{9} \begin{pmatrix} 4 & -4 & 2 \\ -4 & 4 & -2 \\ 2 & -2 & 1 \end{pmatrix}$$

$$(3) P = \begin{pmatrix} \frac{2}{3} & \frac{1}{\sqrt{5}} & \frac{4}{3\sqrt{5}} \\ \frac{-2}{3} & 0 & \frac{5}{3\sqrt{5}} \\ \frac{1}{3} & \frac{-2}{\sqrt{5}} & \frac{2}{3\sqrt{5}} \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 8 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

$$A = 8 \cdot \frac{1}{9} \begin{pmatrix} 4 & -4 & 2 \\ -4 & 4 & -2 \\ 2 & -2 & 1 \end{pmatrix} - 1 \cdot \frac{1}{9} \begin{pmatrix} 5 & 4 & -2 \\ 4 & 5 & 2 \\ -2 & 2 & 8 \end{pmatrix}$$

$$(4) P = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & 0 & 1 \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix},$$

$$A = 3 \cdot \frac{1}{3} \begin{pmatrix} 2 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix} - 3 \cdot \frac{1}{3} \begin{pmatrix} 1 & 1 & -1 & 0 \\ 1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 1 \end{pmatrix}$$

$$3. (1) P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}, \quad A = 0 \cdot \frac{1}{2} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix} + 2 \cdot \frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$$

$$(2) P = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2i}{\sqrt{6}} & \frac{i}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{pmatrix},$$

$$A = 2 \cdot \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} + 6 \cdot \frac{1}{6} \begin{pmatrix} 1 & 2i & -1 \\ -2i & 4 & 2i \\ -1 & -2i & 1 \end{pmatrix} + 3 \cdot \frac{1}{3} \begin{pmatrix} 1 & -i & -1 \\ i & 1 & -i \\ -1 & i & 1 \end{pmatrix}$$

$$(3) P = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2i}{\sqrt{6}} & \frac{i}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

$$A = 1 \cdot \frac{1}{3} \begin{pmatrix} 2 & i & 1 \\ -i & 2 & i \\ 1 & -i & 2 \end{pmatrix} - 2 \cdot \frac{1}{3} \begin{pmatrix} 1 & -i & -1 \\ i & 1 & -i \\ -1 & i & 1 \end{pmatrix}$$

$$(4) P = \frac{1}{2} \begin{pmatrix} 1 & \sqrt{2} & 0 & 1 \\ i & 0 & \sqrt{2} & -i \\ -1 & \sqrt{2} & 0 & -1 \\ -i & 0 & \sqrt{2} & i \end{pmatrix}, \quad P^{-1}AP = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix},$$

$$A = -1 \cdot \frac{1}{4} \begin{pmatrix} 1 & -i & -1 & i \\ i & 1 & -i & -1 \\ -1 & i & 1 & -i \\ -i & -1 & i & 1 \end{pmatrix} + 3 \cdot \frac{1}{4} \begin{pmatrix} 3 & i & 1 & -i \\ -i & 3 & i & 1 \\ 1 & -i & 3 & i \\ i & 1 & -i & 3 \end{pmatrix}$$

$$4. (1) \frac{1}{3} \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix} \vec{x} \quad (2) \vec{x}'$$

$$5. (1) \frac{1}{5} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{pmatrix} \vec{x} \quad (2) \vec{x}' \quad (3) \text{共に } \frac{1}{5}(x_2 + 2x_3)(y_2 + 2y_3)$$

$$6. (1) \frac{e^2}{2} \begin{pmatrix} e + e^{-1} & -(e - e^{-1}) \\ -(e - e^{-1}) & e + e^{-1} \end{pmatrix} \quad (2) e^\alpha \begin{pmatrix} 1 & 1 & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(3) \frac{1}{6} \begin{pmatrix} e + 3e^3 + 2e^{-2} & -2e + 2e^{-2} & 3e + 3e^3 - 6e^{-2} \\ -4e + 6e^3 - 2e^{-2} & 8e - 2e^{-2} & -12e + 6e^3 + 6e^{-2} \\ -2e + 3e^3 - 2e^{-2} & 4e - 2e^{-2} & -6e + 3e^3 + 6e^{-2} \end{pmatrix}$$

宿題 §2

1.

2.

3.

4. (1) 15 (2) $4.8 \times 10^{-15} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} [N]$

5. (1) 2 (2) $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, 4 (4) $\frac{1}{3} \begin{pmatrix} 4 & -1 & 1 \\ -1 & 4 & -1 \\ 1 & -1 & 4 \end{pmatrix}$

6. (4) $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

宿題 §3

1.

2.

3. $\begin{pmatrix} 3t^2 - 4 \\ 2t + 4 \\ 16t - 9t^2 \end{pmatrix}, \begin{pmatrix} 6t \\ 2 \\ 16 - 18t \end{pmatrix}, \frac{16}{3} \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \frac{1}{3} \begin{pmatrix} 4 \\ -26 \\ -44 \end{pmatrix}.$

4. (4) $e = \sqrt{1 + \frac{2}{m} \left(\frac{L}{A} \right)^2 E}$

5. $\operatorname{div} \vec{a} = 2, \operatorname{rot} \vec{a} = \vec{0}_0. \operatorname{div} \vec{a} = 0, \operatorname{rot} \vec{a} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}.$

6. $\begin{pmatrix} 6xy \\ 3x^2 - 3y^2z^2 \\ -2y^3z \end{pmatrix}, 2xz - 6y^2z^2 + xy^2, \begin{pmatrix} 2xyz + 4y^3z \\ x^2 - y^2z \\ 0 \end{pmatrix}.$

7. $\begin{pmatrix} 2xyz \\ x^2z \\ x^2y \end{pmatrix}, 2z^2 - z + 9xz^2, \begin{pmatrix} y \\ 4xz - 3z^3 \\ 0 \end{pmatrix}, \begin{pmatrix} -4x + 9z^2 \\ 0 \\ 4z - 1 \end{pmatrix}, \begin{pmatrix} -2yz^2 \\ -2xz^2 + 3z^4 \\ -4xyz + 12yz^3 \end{pmatrix}.$

8. $\vec{0}, \vec{F}.$

9. $\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \frac{\vec{r}}{r}, \frac{1}{4\pi\epsilon_0} \left(3 \frac{\vec{d} \cdot \vec{r}}{r^5} \vec{r} - \frac{1}{r^3} \vec{d} \right), \frac{\mu_0}{4\pi} \left(3 \frac{\vec{m} \cdot \vec{r}}{r^5} \vec{r} - \frac{1}{r^3} \vec{m} \right).$

10. $\pm \frac{1}{3} \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

11. $2\vec{\omega}$

12.

13. $\vec{k} \cdot \vec{E}_0 = 0, \vec{k} \cdot \vec{B}_0 = 0, \vec{k} \times \vec{E}_0 = \omega \vec{B}_0, \vec{k} \times \vec{B}_0 = -\frac{1}{c^2} \omega \vec{E}_0. (\Rightarrow \vec{k}^2 = \frac{\omega^2}{c^2})$

宿題 §4

法線の向きの指定のない問題では，どちらか一つの向きを選んで計算した値である。

1. $\frac{2}{5}M \frac{a^5 - b^5}{a^3 - b^3}$

2. (1) $\vec{v}_0 + \begin{pmatrix} -\rho\omega \sin \omega t \\ \rho\omega \cos \omega t - \rho\omega \\ 0 \end{pmatrix}, (2) \vec{x}_0 + \vec{v}_0 t + \begin{pmatrix} \rho \cos \omega t - \rho \\ \rho \sin \omega t - \rho\omega t \\ 0 \end{pmatrix}.$

3. (1) (i) $\frac{31}{6}\sqrt{3}$ (ii) $1 + \frac{13}{2}\sqrt{2}$ (iii) $\frac{1}{6}(14\sqrt{14} - 1)$ (iv) 0
 (2) (i) $\frac{7}{2}$ (ii) $\frac{15}{2}$ (iii) $\frac{64}{15}$ (iv) 0
 (3) (i) 2 (ii) 2 (iii) 2 (iv) 0
 (4) 0 (5) $-x^2y - xz^3 + \text{const.}$ (6) 2

4. (1) $\begin{pmatrix} \frac{8}{11} \\ \frac{4}{5} \\ 1 \end{pmatrix}$ (2) $\begin{pmatrix} -\frac{9}{10} \\ -\frac{2}{3} \\ \frac{7}{5} \end{pmatrix}$

5. (1) $\frac{\mu_0 I}{4\pi} \frac{2}{x^2 + y^2} \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$ (2) $\frac{\mu_0 I}{4\pi} \frac{2\pi a^2}{(a^2 + z^2)^{\frac{3}{2}}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

6. (1) 24 (2) 参考：120 = -48 + 60 + 108

7. (1) $\begin{pmatrix} 100 \\ 100 \\ 0 \end{pmatrix}$ (2) 90 (3) 参考：-580 = 4π - 600 + (-4π + 20) + 0

8. 参考：π

9. 参考：2 = 0 + 0 + 0 + 2 - $\frac{1}{2}$ + $\frac{1}{2}$

10. (1) $\frac{2}{15}$ (2) 参考： $\frac{8}{3} = 0 - 1 + 0 + \frac{11}{3}$

11. 参考：84π = 36π + 0 + 48π

宿題 §5

1.

2.