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Fermionic Construction of N=0,1,2 (Super)Conformal Algebras

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Abstract

We derive conditions for fermionic construction of N=0,1 and 2 (super)conformal algebras, and obtain some solutions satisfying these conditions. In the N=2 case these conditions can be simplified when supercurrents are written by complex fermions.

1. Introduction

Recently a great deal of attention has been paid to the conformally and superconformally invariant two-dimensional field theories in connection to string theory and two-dimensional critical phenomena. Unitary representations of the conformal and superconformal algebras give constraints on possible values of critical indices. For N=0,1 and 2 Kac-determinant formulae have been obtained [1-3], and possible values of the central charge c, highest weight and U(1) charge [1,2,4], and corresponding characters [5-8] for the unitary irreducible representation have been derived. Unitary representations of the (super)conformal algebra are realized in several ways (free fermions, free bosons, free fermions and bosons, Z_N-currents and a free boson,...)[2,4,10-18]. It is known that fermionic representation of (super)conformal algebra is relevant to constructing some models which are difficult to be realized by bosons or boson-fermion models by projection[10]. In the case of N=2, representations with $\frac{c}{3}\geq 1$ are also important in their relevance to the compactification of superstring [9-11].

In this paper we study the representations of N=0,1 and 2 (super)conformal algebras constructed out of fermions, which are manifestly unitary. When the supercurrent G(z) is represented as $(\text{fermion})^3$, we derive necessary and sufficient conditions such that it forms superconformal algebra and also give several solutions. We use the method of operator product expansions (OPE) following ref [10]. (We do not restrict the energy momentum tensor to a bilinear form.) In the N=2 case, for instance, we assume $G=\sum(\text{coefficient})\times(\text{fermion})^3$, and calculate the OPE of $G\bar{G}$, which defines the energy momentum tensor T(z) and the U(1) current J(z). The OPE of $T, G, \bar{G}$ and J must realize the N=2 superconformal algebra, so constraints are imposed on the coefficients in G.

We denote free real fermions by $H_a(z), H_\alpha(z), \ldots$, and free complex fermions by $\psi_a(z), \varphi_\alpha(z), \chi_p(z), \ldots$. Propagators are given by $\langle H_a(z)H_b(w)\rangle = \langle \psi_a(z)\bar{\psi}_b(w)\rangle = \delta_{ab}\frac{1}{z-w}$ (Neveu-Schwartz fermion) and Wick's theorem is applied. Repeated indices are summed unless otherwise mentioned. [ ] means antisymmetrization (e.g. $A_{[ab]}=\frac{1}{2}(A_{ab}-A_{ba})$), ( ) means symmetrization and $^-$ denotes complex conjugation.

2. (N=0) conformal algebra

The conformal algebra T(z) is defined by the OPE

$$T(z)T(w)\sim \frac{c}{2(z-w)^4}+\frac{2}{(z-w)^2}T(w)+\frac{1}{z-w}\partial T(w) . \qquad (1)$$

We assume $T(z)=A_{ab}\frac{1}{2}:\partial H_a(z)H_b(z):+A_{abcd}H_aH_bH_cH_d(z)$, where A_{ab} and A_{abcd} are real and $A_{abcd}=A_{[abcd]}$. Then T(z) satisfies the conformal algebra if and only if

$$c=\frac{1}{2}\mathrm{tr}A+\frac{3}{4}\mathrm{tr}A({}^tA-A) ,$$

$$A_{ab}=\frac{1}{2}((A+{}^tA)A)_{ab}-12A_{abcd}A_{cd}+96A_{acde}A_{bcde} , \qquad (2)$$

$$A_{abcd}=2A_{e[a}A_{bcd]e}-36A_{efa[b}A_{cd]ef} .$$

$A_{ab}=\delta_{ab}$, $A_{abcd}=0$ is a solution of (2) and then T(z) is a free energy momentum tensor. In the case where T is the Sugawara form, many solutions are given in [15-17].

3. N=1 superconformal algebra

$\mathcal{O}(z)$ is called a conformal field with conformal dimension h if

$$T(z)\mathcal{O}(w)\sim \frac{h}{(z-w)^2}\mathcal{O}(w)+\frac{1}{z-w}\partial\mathcal{O}(w) . \qquad (3)$$

The N=1 superconformal algebra T(z) and G(z) is defined by (1) and the following OPE

$$G(z)G(w)\sim \frac{2c}{3(z-w)^3}+\frac{2}{z-w}T(w) \qquad (4)$$

and G is a conformal field with $h=\frac{3}{2}$.

We assume $G(z)=-i\eta_{abc}H_aH_bH_c(z)$, where η_{abc} is real and $\eta_{abc}=\eta_{[abc]}$. Since $\eta_{abc}\eta_{a'bc}$ is a positive-semidefinite symmetric matrix, it is diagonalized by some orthogonal matrix. So we can assume $\eta_{abc}\eta_{a'bc}=x_a\delta_{aa'}$ (no sum over a) without loss of generality and $x_a>0$ ($x_a=0$ means $\eta_{abc}=0$ (all b,c), so H_a is decoupled.). From the OPE of GG, T is defined by $T(z)=18x_aL_{H_a}(z) + A_{abcd}H_aH_bH_cH_d(z)$, where $L_{H_a}(z)=\frac{1}{2}:\partial H_aH_a(z):$ (no sum over a) and $A_{abcd}=-\frac{9}{2}\eta_{ea[b}\eta_{cd]e}$. We note that in supersymmetric case A_{abcd} in T(z) is written in terms of η_{abc} in G(z) in contrast to the N=0 case where A_{abcd} is arbitrary. Then T(z) and G(z) satisfy the N=1 superconformal algebra if and only if

$$\frac{2}{3}c=6\sum_a x_a ,$$

$$\eta_{abc}=18(x_a+x_b+x_c)\eta_{abc}-72\eta_{ade}\eta_{bdf}\eta_{cef} \quad \text{(no sum over a,b,c)}. \qquad (5)$$

The right-hand side of the second equation of (5) is equal to $6(x_a+x_b+x_c)\eta_{abc}-24A_{de[ab}\eta_{c]de}$ identically.

A series of solutions of (5) are given by

$$\eta_{abc}=Af_{abc}\ ,\quad \eta_{a\alpha\beta}=BM^a{}_{\alpha\beta} \tag{6}$$

where f_{abc} is the structure constant of a simple Lie algebra $(a=1,\dots,d_G,\ f_{acd}f_{bcd}=c_{adj}\delta_{ab})$ and $M^a{}_{\alpha\beta}$ is a real antisymmetric matrix $(\alpha=1,\dots,d_\lambda)$ and a representation of the Lie algebra,

$$[M^a,M^b]=f_{abc}M^c\ ,\ (\mathrm{tr}M^aM^b=-\kappa_\lambda\delta_{ab}\ ,\ (M^aM^a)_{\alpha\beta}=-c_\lambda\delta_{\alpha\beta})\ . \tag{7}$$

The second equation of (6) means $\eta_{a\alpha\beta}=\eta_{\alpha\beta a}=\eta_{\beta a\alpha}=-\eta_{\alpha a\beta}=-\eta_{\beta\alpha a}=-\eta_{a\beta\alpha}=BM^a{}_{\alpha\beta}$. $A=\frac{1}{3\sqrt{2c_{adj}}}$ and $B=0$ gives $\frac{2}{3}c=\frac{d_G}{3}$ and T is free. $A=\frac{\kappa_\lambda}{3\sqrt{2c_{adj}(\kappa_\lambda+c_{adj})(\kappa_\lambda+2c_{adj})}}$ and $B=\frac{c_{adj}}{3\sqrt{2c_{adj}(\kappa_\lambda+c_{adj})(\kappa_\lambda+2c_{adj})}}$ gives $\frac{2}{3}c=\frac{\kappa_\lambda(\kappa_\lambda+3c_{adj})d_G}{3(\kappa_\lambda+c_{adj})(\kappa_\lambda+2c_{adj})}$ and T is given as a difference of the Sugawara forms[18]. $A=0$ and $B=\frac{1}{3\sqrt{2(\kappa_\lambda+2c_{adj})}}$ gives $\frac{2}{3}c=\frac{\kappa_\lambda d_G}{\kappa_\lambda+2c_{adj}}$ and T is written in terms of currents.

4. N=2 superconformal algebra

N=2 superconformal algebra $T(z), G^1(z), G^2(z)$ and $J(z)$ is defined by (1) and the following OPE

$$G^i(z)G^j(w)\sim \delta^{ij}\left(\frac{2c}{3(z-w)^3}+\frac{2}{z-w}T(w)\right)+i\varepsilon^{ij}\left(\frac{2}{(z-w)^2}J(w)+\frac{1}{z-w}\partial J(w)\right),$$

$$J(z)J(w)\sim\frac{k}{(z-w)^2}\quad (k=\tfrac{c}{3})\ ,\qquad (\varepsilon^{12}=-\varepsilon^{21}=1) \tag{8}$$

$$J(z)G^i(w)\sim\frac{1}{z-w}i\varepsilon^{ij}G^j(w),$$

and $G^i(z)$ and $J(z)$ are conformal fields with $h=\frac{3}{2}$ and 1, respectively. Or in the complex notation $\left.\begin{matrix}G(z)\\ \bar{G}(z)\end{matrix}\right\}=\frac{1}{\sqrt{2}}(G^1\pm iG^2)(z)$, which are conformal fields with $h=\frac{3}{2}$, and

$$G(z)\bar{G}(w)\sim\frac{2c}{3(z-w)^3}+\frac{2}{(z-w)^2}J(w)+\frac{1}{z-w}\partial J(w)+\frac{2}{z-w}T(w)$$

$$G(z)G(w)\sim 0\qquad (\bar{G}(z)\bar{G}(w)\sim 0) \tag{9}$$

$$J(z)G(w)\sim\frac{1}{z-w}G(w)\qquad (J(z)\bar{G}(w)\sim-\frac{1}{z-w}\bar{G}(w))\ .$$

We assume $G^i(z)=-i\eta^{(i)}_{abc}H_aH_bH_c(z)$ $(i=1,2)$, where $\eta^{(i)}_{abc}$ is real and $\eta^{(i)}_{abc}=\eta^{(i)}_{[abc]}$. This is the most general form of (fermion)^3. We can assume $\eta^{(1)}_{abc}\eta^{(1)}_{a'bc}=x_a\delta_{aa'}$ (no sum over a) and $x_a>0$ as before. From the OPE of G^iG^i, T and J are defined by $T(z)=18x_aL_{H_a}(z)+A_{abcd}H_aH_bH_cH_d(z)$, $J(z)=A_{ab}H_aH_b(z)$ where $A_{abcd}=-\frac{9}{2}\eta^{(1)}_{ea[b}\eta^{(1)}_{cd]e}$ and $A_{ab}=-9i\eta^{(1)}_{cd[a}\eta^{(2)}_{b]cd}$. Then G^i, T, J satisfy the N=2 superconformal algebra if and only if

$$\frac{c}{3}=3\sum_a x_a=2\mathrm{tr}A^2$$

$$\eta^{(2)}_{abc}\eta^{(2)}_{a'bc}=x_a\delta_{aa'}\ ,\quad \eta^{(1)}_{ea[b}\eta^{(1)}_{cd]e}=\eta^{(2)}_{ea[b}\eta^{(2)}_{cd]e}$$

$$\eta^{(1)}_{e[ab}\eta^{(2)}_{cd]e}=0\ ,\quad \eta^{(1)}_{cd(a}\eta^{(2)}_{b)cd}=0,$$

$$\eta^{(i)}_{abc}+6i\varepsilon^{ij}A_{d[a}\eta^{(j)}_{bc]d}=0, \tag{10}$$

$$\eta^{(i)}_{abc}A_{bc}=0\ ,\quad A_{e[abc}A_{d]e}=0,$$

$$A_{ab}=9(x_a+x_b)A_{ab}+12A_{abcd}A_{cd}\quad (\text{no sum over } a,b),$$

$$(x_a-x_b)A_{ab}=0\quad (\text{no sum over } a,b)$$

in addition to the condition (5) for $\eta^{(i)}_{abc}$. It is hard to find solutions to eqs. (10) and (5). So we shall use complex fermions, which are suited for the N=2 algebra.

We assume $G(z)=-i\eta_{abc}\psi_a\psi_b\psi_c(z)$ where η_{abc} is complex and $\eta_{abc}=\eta_{[abc]}$. Since $\eta_{abc}\bar{\eta}_{a'bc}$ is a positive-semidefinit hermitian matrix, it is diagonalized by some unitary matrix. So we can assume $\eta_{abc}\bar{\eta}_{a'bc}=x_a\delta_{aa'}$(no sum over a) and $x_a>0$ as before. In this form of the complex fermions the OPE of GG is trivially satisfied. From the OPE of G and $\bar{G}(z)=-i\bar{\eta}_{abc}\bar{\psi}_a\bar{\psi}_b\bar{\psi}_c(z)$. T and J are defined by $T(z)=9x_aL_{\psi_a}(z)-\frac{9}{2}\eta_{abc}\bar{\eta}_{ab'c'}:\psi_b\psi_c\bar{\psi}_{b'}\bar{\psi}_{c'}(z):$ and $J(z)=9x_a:\psi_a\bar{\psi}_b(z):$ where $L_{\psi_a}(z)=\frac{1}{2}:(\partial\psi_a\bar{\psi}_a-\psi_a\partial\bar{\psi}_a)(z):$(no sum over a). We can verify that T, G, $\bar{G}$ and J satisfy the N=2 superconformal algebra if and only if

$$\frac{c}{3}=3\sum_a x_a=81\sum_a x_a^2 \qquad (11)$$

$$\eta_{abc}=9(x_a+x_b+x_c)\eta_{abc} \quad \text{(no sum over a, b, c)}$$

from straightforward calculations. This condition is much simpler than (10). The first equation of (11) comes from the relation between centers of the conformal and Kac-Moody algebra, i.e., $k=\frac{c}{3}$. The second equation of (11) means that G has U(1) charge 1. Again (6) are solutions of (11). $A=\frac{\sqrt{2d_G-d_\lambda}}{3\sqrt{6d_G c_{adj}}}$ and $B=\frac{1}{3\sqrt{6c_\lambda}}$ (we assume $2d_G\geq d_\lambda$) gives $\frac{c}{3}=\frac{d_G+d_\lambda}{9}$. $A=\frac{1}{3\sqrt{3c_{adj}}}$ and B=0 gives $\frac{c}{3}=\frac{d_G}{9}$

and T is $\sum_a L_{\psi_a}$ - Sugawara form. This solution is given in refs.[2] and [13]. For A=0, see the next paragraph.

As a special case of $\eta\psi\psi\psi$, we take $G(z)=-i\eta_{\alpha\beta a}\varphi_\alpha\varphi_\beta\psi_a(z)$ where $\eta_{\alpha\beta a}$ is complex (a=1,...,n, α=1,...,m) and $\eta_{\alpha\beta a}=\eta_{[\alpha\beta]a}$. We can take $\eta_{\alpha\beta a}\bar{\eta}_{\alpha\beta a'}=x_a\delta_{aa'}$(no sum over a), $\eta_{\alpha\beta a}\bar{\eta}_{\alpha'\beta a}=y_\alpha\delta_{\alpha\alpha'}$(no sum over α), $x_a, y_\alpha>0$. (11) reduces to

$$\frac{c}{3}=\sum_a x_a=\sum_\alpha y_\alpha=\Sigma x_a^2+4\sum_\alpha y_\alpha^2 \qquad (12)$$

$$\eta_{\alpha\beta a}=(x_a+2(y_\alpha+y_\beta))\eta_{\alpha\beta a} \quad \text{(no sum over a, }\alpha,\beta\text{)}.$$

If $x_a(y_\alpha)$ are independent of $a(\alpha)$, i.e. if there exists an $\eta_{\alpha\beta a}$ satisfying

$$\eta_{\alpha\beta a}\bar{\eta}_{\alpha\beta a'}=\frac{m}{4n+m}\delta_{aa'}, \quad \eta_{\alpha\beta a}\bar{\eta}_{\alpha'\beta a}=\frac{n}{4n+m}\delta_{\alpha\alpha'} \qquad (13)$$

then it is a solution of (12) and gives

$$\frac{c}{3}=\frac{nm}{4n+m} \qquad (14)$$

From (13) n and m must satisfy $n\leq\frac{1}{2}m(m-1)$. For example, $\eta_{\alpha\beta a}=\frac{1}{\sqrt{\kappa_\lambda+4c_\lambda}}M^a_{\alpha\beta}$ (M^a is a real antisymmetric matrix satisfying (7)) gives $\frac{c}{3}=\frac{d_G d_\lambda}{4d_G+d_\lambda}$ (In particular $\eta_{bca}=\frac{-1}{\sqrt{5c_{adj}}}f_{abc}$ gives $\frac{c}{3}=\frac{d_G}{5}$.). For n=1, we can show that m=even and $\eta_{\alpha\beta 1}=\frac{1}{\sqrt{m+4}}\times$(antisymmetric unitary matrix)$_{\alpha\beta}$ by simple considerations.

$$\eta_{\alpha\beta 1}=\frac{1}{\sqrt{m+4}}\begin{pmatrix} 0 & 1_{\frac{m}{2}} \\ -1_{\frac{m}{2}} & 0 \end{pmatrix}_{\alpha\beta}$$

gives all discrete series $\frac{c}{3}=1-\frac{1}{\frac{m}{2}+2}$. Di Vecchia et. al. gave fermionic costruction of N=2 supercurrent algebra using SU(2) and U(1) currents.[4] And it is this case. For n=2 and m=even(>2).

$$\eta_{\alpha\beta 1}=\frac{1}{\sqrt{m+8}}\begin{pmatrix} 0 & 1_{\frac{m}{2}} \\ -1_{\frac{m}{2}} & 0 \end{pmatrix}_{\alpha\beta} \qquad \eta_{\alpha\beta 2}=\frac{1}{\sqrt{m+8}}\begin{pmatrix} i\sigma_2 & & & 0 \\ 0 & \ddots & ⋰ & i\sigma_2 \\ & ⋰ & \ddots & \\ 0 & i\sigma_2 & & 0 \end{pmatrix}_{\alpha\beta}$$

gives $\frac{c}{3}=\frac{2m}{m+8}$.

As a special case of $\eta\varphi\varphi\psi$, we take $G(z)=-i\eta_{a\alpha p}\psi_a\varphi_\alpha\chi_p(z)$ where $\eta_{a\alpha p}$ is complex ($a=1,\ldots,n$, $\alpha=1,\ldots,m$, $p=1,\ldots,l$). We can take $\eta_{a\alpha p}\bar{\eta}_{a'\alpha p}=x_a\delta_{aa'}$ (no sum over a), $\eta_{a\alpha p}\bar{\eta}_{a\alpha' p}=y_\alpha\delta_{\alpha\alpha'}$ (no sum over α), $\eta_{a\alpha p}\bar{\eta}_{a\alpha p'}=z_p\delta_{pp'}$ (no sum over β) and $x_a, y_\alpha, z_p>0$. (11) reduces to

$$\frac{c}{3}=\frac{1}{2}\sum_a x_a=\frac{1}{2}\sum_\alpha y_\alpha=\frac{1}{2}\sum_p z_p=\frac{1}{4}\left(\sum_a x_a^2+\sum_\alpha y_\alpha^2+\sum_p z_p^2\right) \qquad (15)$$

$$\eta_{a\alpha p}=\frac{1}{2}(x_a+y_\alpha+z_p)\eta_{a\alpha p} \quad \text{(no sum over } a,\alpha,\beta).$$

If $x_a(y_\alpha, z_p)$ are independent of $a(\alpha, p)$, i.e. if there exists an $\eta_{a\alpha p}$ satisfying

$$\eta_{a\alpha p}\bar{\eta}_{a'\alpha p}=\frac{2ml}{nm+ml+ln}\delta_{aa'}, \quad \eta_{a\alpha p}\bar{\eta}_{a\alpha' p}=\frac{2ln}{nm+ml+ln}\delta_{\alpha\alpha'}, \qquad (16)$$

$$\eta_{a\alpha p}\bar{\eta}_{a\alpha p'}=\frac{2nm}{nm+ml+ln}\delta_{pp'},$$

then it is a solution of (15) and gives

$$c=\frac{3}{\frac{1}{n}+\frac{1}{m}+\frac{1}{l}} \qquad (17)$$

From (16) n, m and l must satisfy $n\leq ml$, $m\leq ln$ and $l\leq nm$. For example, for $n=d_G$, $m=l=d_\lambda$, $\eta_{a\alpha\beta}=\frac{\sqrt{2}}{\sqrt{\kappa_\lambda+2c_\lambda}}M^a{}_{\alpha\beta}$ gives $\frac{c}{3}=\frac{d_G d_\lambda}{2d_G+d_\lambda}$ where M^a is an antihermitian matrix satisfying (7). Especially for $n=m=l=d_G$, $\eta_{abc}=-\frac{\sqrt{2}}{\sqrt{3c_{adj}}}f_{abc}$ gives $\frac{c}{3}=\frac{d_G}{3}$. For $n=m=l$, $\eta_{abc}=\frac{\sqrt{2}}{\sqrt{3n^3}}\sum_{k=0}^{n-1}\exp(2\pi i\frac{k}{n}(a+b+c-3))=\frac{\sqrt{2}}{\sqrt{3n}}\sum_{k=0}^{2}\delta_{a+b+c-3,kn}$ gives $\frac{c}{3}=\frac{n}{3}$. When m and l are relatively prime and $n=ml$, $\eta_{a\alpha p}=\frac{1}{n}\frac{\sqrt{2}}{\sqrt{m+l+1}}\sum_{k=0}^{n-1}\exp(2\pi i\frac{k}{n}(a-1+l(\alpha-1)+m(p-1)))$ gives $\frac{c}{3}=\frac{ml}{m+l+1}$. If $l=1$, then $n=m$, $\eta_{ab1}=\frac{\sqrt{2}}{\sqrt{n+2}}\delta_{ab}$ and $\frac{c}{3}=1-\frac{2}{n+2}$, which was obtained in the previous paragraph. For $l=2$ and $n=m\geq 2$, $\eta_{ab1}=\frac{\sqrt{2}}{\sqrt{n+4}}\delta_{ab}$, $\eta_{ab2}=\frac{\sqrt{2}}{\sqrt{n+4}}(\text{traceless unitary matrix})_{ab}$ gives $\frac{c}{3}=\frac{2n}{n+4}$. In the $l=2$ and $n\neq m$ case, for instance, $\eta_{a\alpha\beta}=\frac{1}{\sqrt{5}}\sigma^a_{\alpha\beta}$ ($\sigma^4_{\alpha\beta}=\delta_{\alpha\beta}$, $m=2, n=4$) gives a solution with $\frac{c}{3}=\frac{4}{5}$.

Now $G(z)=-i\eta_{abc}:\bar{\psi}_a\psi_b\psi_c(z):$ (where η_{abc} is complex and $\eta_{abc}=\eta_{a[bc]}$) is left to be considered. In general, OPE of GG is nontrivial and the final condition is as complicated as (10), so we shall not write it down in this paper. But for a special form $G(z)=-i\eta_{\alpha\beta a}:\bar{\varphi}_\alpha\varphi_\beta:\psi_a(z)$ ($\eta_{\alpha\beta a}$ is complex), it is easy to show that $\frac{c}{3}$=integer. $G(z)=\frac{1}{\sqrt{2}}(:\bar{\varphi}_1\varphi_1: +i:\bar{\varphi}_2\varphi_2:)\psi(z)$, which can be given by fermionization of $G=i\partial\phi\psi(z)$ (ϕ is a complex boson), is an example of this case.

5. conclusions and discussions

In this paper we discussed fermionic construction of N=0.1 and 2 (super)conformal algebra. Starting from general form of supercurrents. we derived conditions for them to realize conformal algebra. In particular. it was shown that. in N=2 case, these conditions reduced to a simpler form when we use complex fermions. We gave several solutions which satisfy these conditions. Our construction will give a step forward in understanding and studying the structure of superconformal algebras.

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