Fermionic Construction of N=0,1,2 (Super)Conformal Algebras*

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Abstract

We derive conditions for fermionic construction of N=0,1 and 2 (super)conformal algebras, and obtain some solutions satisfying these conditions. In the N=2 case these conditions can be simplified when supercurrents are written by complex fermions.

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§1. Introduction

Recently a great deal of attention has been paid to the conformally and superconformally invariant two-dimensional field theories in connection to string theory and two-dimensional critical phenomena. Unitary representations of the (super)conformal algebra ,which give constraints on possible values of the central charge c, highest weight and U(1) charge [1,2,4], are realized in several ways (free fermions, free bosons, free fermions and bosons, Z_N-currents and a free boson,...)[2,4,6-14]. It is known that fermionic representation of (super)conformal algebra is relevant to constructing some models which are difficult to be realized by bosons or boson-fermion models by projection[6]. In the case of N=2, representations with $\frac{c}{3}\geq 1$ are also important in their relevance to the compactification of superstring [5-7].

In this talk we study the representations of N=0,1 and 2 (super)conformal algebras constructed out of fermions, which are manifestly unitary [15]. We use the method of operator product expansions (OPE) following ref [6]. (We do not restrict the energy momentum tensor to a bilinear form.) In the N=2 case, for instance, we assume $G=\sum(\text{coefficient})\times(\text{fermion})^3$, and calculate the OPE of $G\bar{G}$, which defines the energy momentum tensor T(z) and the U(1) current J(z). The OPE of $T, G, \bar{G}$ and J must realize the N=2 superconformal algebra, so constraints are imposed on the coefficients in G.

We denote free real fermions by $H_a(z), H_\alpha(z), \ldots$, and free complex fermions by $\psi_a(z), \varphi_\alpha(z), \chi_p(z), \ldots$. Propagators are given by $\langle H_a(z)H_b(w)\rangle = \langle\psi_a(z)\bar{\psi}_b(w)\rangle = \delta_{ab}\ \frac{1}{z-w}$ (Neveu-Schwartz fermion) and Wick's theorem is applied. Repeated indices are summed unless otherwise mentioned. [ ] means antisymmetrization (e.g. $A_{[ab]}=\frac{1}{2}(A_{ab}-A_{ba})$) , ( ) means symmetrization and $\bar{\ }$ denotes complex conjugation.

§2. N=0,1(super)conformal algebras

The conformal (Virasoro) algebra T(z) is defined by the OPE

$$T(z)T(w)\sim \frac{c}{2(z-w)^4}+\frac{2}{(z-w)^2}T(w)+\frac{1}{z-w}\partial T(w)\ . \tag{1}$$

We assume $T(z)=A_{ab}\frac{1}{2}:\partial H_a(z)H_b(z):+A_{abcd}H_aH_bH_cH_d(z)$, where A_{ab} and A_{abcd} are real and $A_{abcd}=A_{[abcd]}$. Then T(z) satisfies the conformal algebra if and only if

$$\begin{aligned}&c=\frac{1}{2}\mathrm{tr}A+\frac{3}{4}\mathrm{tr}A({}^tA-A)\ ,\\&A_{ab}=\frac{1}{2}((A+{}^tA)A)_{ab}-12A_{abcd}A_{cd}+96A_{acde}A_{bcde}\ ,\\&A_{abcd}=2A_{e[a}A_{bcd]e}-36A_{efa[b}A_{cd]ef}\ .\end{aligned}\tag{2}$$

Free energy momentum tensor($A_{ab}=\delta_{ab}$, $A_{abcd}=0$) and Sugawara form [11-13] are solutions of (2)

$\mathcal{O}(z)$ is called a conformal (primary) field with conformal dimension h if

$$T(z)\mathcal{O}(w)\sim \frac{h}{(z-w)^2}\mathcal{O}(w)+\frac{1}{z-w}\partial\mathcal{O}(w)\ . \tag{3}$$

The N=1 superconformal algebra T(z) and G(z) is defined by (1) and the following OPE

$$G(z)G(w)\sim \frac{2c}{3(z-w)^3}+\frac{2}{z-w}T(w) \tag{4}$$

and G is a conformal field with $h=\frac{3}{2}$.

We assume $G(z)=-i\eta_{abc}H_aH_bH_c(z)$, where η_{abc} is real and $\eta_{abc}=\eta_{[abc]}$. Since $\eta_{abc}\eta_{a'bc}$ is a positive-semidefinite symmetric matrix, it is diagonalized by some orthogonal matrix. So we can assume $\eta_{abc}\eta_{a'bc}=x_a\delta_{aa'}$ (no sum over a) without loss of generality and $x_a>0$ ($x_a=0$ means $\eta_{abc}=0$ (all b,c), so H_a is decoupled.). From the OPE of GG, T is defined by $T(z)=18x_aL_{H_a}(z) + A_{abcd}H_aH_bH_cH_d(z)$, where $L_{H_a}(z)=\frac{1}{2}:\partial H_aH_a(z):$ (no sum over a) and $A_{abcd}=-\frac{9}{2}\eta_{ea[b}\eta_{cd]e}$. We note that in supersymmetric case A_{abcd} in T(z) is written in terms of η_{abc} in G(z) in contrast to the N=0 case where A_{abcd} is arbitrary. Then T(z) and G(z) satisfy the N=1 superconformal algebra if and only if

$$\frac{2}{3}c=6\sum_a x_a,$$

$$\eta_{abc}=18(x_a+x_b+x_c)\eta_{abc}-72\eta_{ade}\eta_{bdf}\eta_{cef} \quad \text{(no sum over a,b,c)}. \qquad (5)$$

The right-hand side of the second equation of (5) is equal to $6(x_a+x_b+x_c)\eta_{abc}-24A_{de[ab}\eta_{c]de}$ identically. Solutions of (5) are found in [14,15].

§3. N=2 superconformal algebra

N=2 superconformal algebra $T(z),G^1(z),G^2(z)$ and $J(z)$ is defined by (1) and the following OPE

$$G^i(z)G^j(w)\sim \delta^{ij}\left(\frac{2c}{3(z-w)^3}+\frac{2}{z-w}T(w)\right)+i\varepsilon^{ij}\left(\frac{2}{(z-w)^2}J(w)+\frac{1}{z-w}\partial J(w)\right),$$

$$J(z)J(w)\sim \frac{k}{(z-w)^2} \quad (k=\frac{c}{3}) \;, \qquad (\varepsilon^{12}=-\varepsilon^{21}=1) \tag{6}$$

$$J(z)G^i(w)\sim \frac{1}{z-w} i\varepsilon^{ij}G^j(w),$$

and $G^i(z)$ and $J(z)$ are conformal fields with $h=\frac{3}{2}$ and 1, respectively. Or in the complex notation $\left.\begin{matrix}G(z)\\ \bar{G}(z)\end{matrix}\right\}=\frac{1}{\sqrt{2}}(G^1\pm iG^2)(z)$, which are conformal fields with $h=\frac{3}{2}$, and

$$G(z)\bar{G}(w)\sim \frac{2c}{3(z-w)^3}+\frac{2}{(z-w)^2}J(w)+\frac{1}{z-w}\partial J(w)+\frac{2}{z-w}T(w)$$

$$G(z)G(w)\sim 0 \qquad (\bar{G}(z)\bar{G}(w)\sim 0) \tag{7}$$

$$J(z)G(w)\sim \frac{1}{z-w}G(w) \qquad (J(z)\bar{G}(w)\sim -\frac{1}{z-w}\bar{G}(w)) \;.$$

We assume $G^i(z)=-i\eta^{(i)}_{abc}H_aH_bH_c(z)$ $(i=1,2)$, where $\eta^{(i)}_{abc}$ is real and $\eta^{(i)}_{abc}=\eta^{(i)}_{[abc]}$. This is the most general form of (fermion)^3. We can assume $\eta^{(1)}_{abc}\eta^{(1)}_{a'bc}=x_a\delta_{aa'}$,(no sum over a) and $x_a>0$ as before. From the OPE of G^1G^i, T and J are defined by $T(z)=18x_aL_{H_a}(z)+A_{abcd}H_aH_bH_cH_d(z)$, $J(z)=A_{ab}H_aH_b(z)$ where $A_{abcd}=-\frac{9}{2}\eta^{(1)}_{ea[b}\eta^{(1)}_{cd]e}$ and $A_{ab}=-9i\eta^{(1)}_{cd[a}\eta^{(2)}_{b]cd}$. Then G^i,T,J satisfy the N=2 superconformal algebra if and only if

$$\frac{c}{3}=3\sum_a x_a=2\,\mathrm{tr}A^2$$

$$\eta^{(2)}_{abc}\eta^{(2)}_{a'bc}=x_a\delta_{aa'},\ \text{(no sum over a)}\ ,\quad \eta^{(1)}_{ea[b}\eta^{(1)}_{cd]e}=\eta^{(2)}_{ea[b}\eta^{(2)}_{cd]e}$$

$$\eta^{(1)}_{e[ab}\eta^{(2)}_{cd]e}=0 \qquad , \quad \eta^{(1)}_{cd(a}\eta^{(2)}_{b)cd}=0,$$

$$\eta^{(i)}_{abc}+6i\varepsilon^{ij}A_{d[a}\eta^{(j)}_{bc]d}=0, \tag{8}$$

$$\eta^{(i)}_{abc}A_{bc}=0 \qquad , \quad A_{e[abc}A_{d]e}=0,$$

$$A_{ab}=9(x_a+x_b)A_{ab}+12A_{abcd}A_{cd} \quad \text{(no sum over a,b)},$$

$$(x_a-x_b)A_{ab}=0 \quad \text{(no sum over a,b)}$$

in addition to the condition (5) for $\eta^{(i)}_{abc}$. It is hard to find solutions to eqs. (8) and (5). So we shall use complex fermions, which are suited for the N=2 algebra.

We assume $G(z)=-i\eta_{abc}\psi_a\psi_b\psi_c(z)$ where η_{abc} is complex and $\eta_{abc}=\eta_{[abc]}$. Since $\eta_{abc}\bar{\eta}_{a'bc}$ is a positive-semidefinit hermitian matrix, it is diagonalized by some unitary matrix. So we can assume $\eta_{abc}\bar{\eta}_{a'bc}=x_a\delta_{aa'}$(no sum over a) and $x_a>0$ as before. In this form of the complex fermions the OPE of GG is trivially satisfied. From the OPE of G and $\bar{G}(z)=-i\bar{\eta}_{abc}\bar{\psi}_a\bar{\psi}_b\bar{\psi}_c(z)$, T and J are defined by $T(z)=9x_aL_{\psi_a}(z)-\frac{9}{2}\eta_{abc}\bar{\eta}_{ab'c'}:\psi_b\psi_c\bar{\psi}_{b'}\bar{\psi}_{c'}(z):$ and $J(z)=9x_a:\psi_a\bar{\psi}_a(z):$ where $L_{\psi_a}(z)=\frac{1}{2}:(\partial\psi_a\bar{\psi}_a-\psi_a\partial\bar{\psi}_a)(z):$(no sum over a). We can verify that T,G,$\bar{G}$ and J satisfy the N=2 superconformal algebra if and only if

$$\frac{c}{3}=3\sum_a x_a=81\sum_a x_a^2 \quad , \qquad (9)$$

$$\eta_{abc}=9(x_a+x_b+x_c)\eta_{abc} \quad \text{(no sum over a, b, c)}$$

from straightforward calculations. This condition is much simpler than (8). The first equation of (9) comes from the relation between centers of the conformal and Kac-Moody algebra, i.e., $k=\frac{c}{3}$. The second equation of (9) means that G has U(1) charge 1. A series of solutions of (9) are given by

$$\eta_{abc}=Af_{abc}\ ,\quad \eta_{a\alpha\beta}=BM^{a}{}_{\alpha\beta} \tag{10}$$

where f_{abc} is the structure constant of a simple Lie algebra ($a=1,\dots,d_G$, $f_{acd}f_{bcd}=c_{adj}\delta_{ab}$) and $M^a{}_{\alpha\beta}$ is a real antisymmetric matrix ($\alpha=1,\dots,d_\lambda$) and a representation of the Lie algebra,

$$[M^a,M^b]=f_{abc}M^c\ ,\ (\mathrm{tr}M^aM^b=-\kappa_\lambda\delta_{ab}\ ,\ (M^aM^a)_{\alpha\beta}=-c_\lambda\delta_{\alpha\beta})\ . \tag{11}$$

The second equation of (6) means $\eta_{a\alpha\beta}=\eta_{\alpha\beta a}=\eta_{\beta a\alpha}=-\eta_{\alpha a\beta}=-\eta_{\beta\alpha a}=-\eta_{a\beta\alpha}=BM^a{}_{\alpha\beta}$. $A=\frac{\sqrt{2d_G-d_\lambda}}{3\sqrt{6d_Gc_{adj}}}$ and $B=\frac{1}{3\sqrt{6c_\lambda}}$ (we assume $2d_G\geq d_\lambda$) gives $\frac{c}{3}=\frac{d_G+d_\lambda}{9}$. $A=\frac{1}{3\sqrt{3c_{adj}}}$ and $B=0$ gives $\frac{c}{3}=\frac{d_G}{9}$ and T is $\sum_a L_{\psi_a}$ - Sugawara form. This solution is given in refs.[2] and [9]. For A=0, see the next paragraph.

As a special case of $\eta\psi\psi\psi$, we take $G(z)=-i\eta_{\alpha\beta a}\varphi_\alpha\varphi_\beta\psi_a(z)$ where $\eta_{\alpha\beta a}$ is complex ($a=1,\dots,n$, $\alpha=1,\dots,m$) and $\eta_{\alpha\beta a}=\eta_{[\alpha\beta]a}$. We can take $\eta_{\alpha\beta a}\bar{\eta}_{\alpha\beta a'}=x_a\delta_{aa'}$,(no sum over a), $\eta_{\alpha\beta a}\bar{\eta}_{\alpha'\beta a}=y_\alpha\delta_{\alpha\alpha'}$,(no sum over α), $x_a,y_\alpha>0$. (9) reduces to

$$\frac{c}{3}=\sum_a x_a=\sum_\alpha y_\alpha=\Sigma x_a^2+4\sum_\alpha y_\alpha^2 \tag{12}$$

$$\eta_{\alpha\beta a}=(x_a+2(y_\alpha+y_\beta))\eta_{\alpha\beta a}\quad (\text{no sum over } a,\alpha,\beta).$$

If $x_a(y_\alpha)$ are independent of $a(\alpha)$, i.e. if there exists an $\eta_{\alpha\beta a}$ satisfying

$$\eta_{\alpha\beta a}\bar{\eta}_{\alpha\beta a'}=\frac{m}{4n+m}\delta_{aa'}\quad ,\quad \eta_{\alpha\beta a}\bar{\eta}_{\alpha'\beta a}=\frac{n}{4n+m}\delta_{\alpha\alpha'} \tag{13}$$

then it is a solution of (12) and gives

$$\frac{c}{3}=\frac{nm}{4n+m} \quad . \tag{14}$$

From (13) n and m must satisfy $n\leq\frac{1}{2}m(m-1)$. For example, $\eta_{\alpha\beta a}=\frac{1}{\sqrt{\kappa_\lambda+4c_\lambda}}M^a_{\alpha\beta}$ (M^a is a real antisymmetric matrix satisfying (11)) gives $\frac{c}{3}=\frac{d_G d_\lambda}{4d_G+d_\lambda}$ (In particular $\eta_{bca}=\frac{-1}{\sqrt{5c_{adj}}}f_{abc}$ gives $\frac{c}{3}=\frac{d_G}{5}$.). For n=1, we can show that m=even and $\eta_{\alpha\beta 1}=\frac{1}{\sqrt{m+4}}\times$(antisymmetric unitary matrix)$_{\alpha\beta}$ by simple considerations.

$$\eta_{\alpha\beta 1}=\frac{1}{\sqrt{m+4}}\begin{pmatrix} 0 & I_{m/2} \\ -I_{m/2} & 0 \end{pmatrix}_{\alpha\beta}$$

gives all discrete series $\frac{c}{3}=1-\frac{1}{\frac{m}{2}+2}$. Di Vecchia et. al. gave fermionic costruction of N=2 supercurrent algebra using SU(2) and U(1) currents.[4] And it is this case.

As a special case of $\eta\varphi\varphi\psi$, we take $G(z)=-i\eta_{a\alpha p}\psi_a\varphi_\alpha\chi_p(z)$ where $\eta_{a\alpha p}$ is complex ($a=1,..,n$, $\alpha=1,..,m$, $p=1,..,l$). We can take $\eta_{a\alpha p}\bar{\eta}_{a'\alpha p}=x_a\delta_{aa'}$,(no sum over a), $\eta_{a\alpha p}\bar{\eta}_{a\alpha' p}=y_\alpha\delta_{\alpha\alpha'}$,(no sum over α), $\eta_{a\alpha p}\bar{\eta}_{a\alpha p'}=z_p\delta_{pp'}$,(no sum over β) and $x_a,y_\alpha,z_p>0$. (9) reduces to

$$\frac{c}{3}=\frac{1}{2}\sum_a x_a=\frac{1}{2}\sum_\alpha y_\alpha=\frac{1}{2}\sum_p z_p=\frac{1}{4}\left(\sum_a x_a^2+\sum_\alpha y_\alpha^2+\sum_p z_p^2\right) \quad , \tag{15}$$

$$\eta_{a\alpha p}=\frac{1}{2}(x_a+y_\alpha+z_p)\eta_{a\alpha p} \quad \text{(no sum over } a,\alpha,\beta).$$

If $x_a(y_\alpha,z_p)$ are independent of $a(\alpha,p)$, i.e. if there exists an $\eta_{a\alpha p}$ satisfying

$$\eta_{a\alpha p}\bar{\eta}_{a'\alpha p}=\frac{2ml}{nm+ml+ln}\delta_{aa'}\ ,\ \eta_{a\alpha p}\bar{\eta}_{a\alpha' p}=\frac{2ln}{nm+ml+ln}\delta_{\alpha\alpha'} \qquad (16)$$

$$\eta_{a\alpha p}\bar{\eta}_{a\alpha p'}=\frac{2nm}{nm+ml+ln}\delta_{pp'}$$

then it is a solution of (15) and gives

$$c=\frac{3}{\frac{1}{n}+\frac{1}{m}+\frac{1}{l}} \ . \qquad (17)$$

From (16) n,m and l must satisfy $n\leq ml$, $m\leq ln$ and $l\leq nm$. For example, for $n=d_G$, $m=l=d_\lambda$, $\eta_{a\alpha\beta}=\frac{\sqrt{2}}{\sqrt{\kappa_\lambda+2c_\lambda}}M^a{}_{\alpha\beta}$ gives $\frac{c}{3}=\frac{d_G d_\lambda}{2d_G+d_\lambda}$ where M^a is an antihermitian matrix satisfying (11). Especially for $n=m=l=d_G$, $\eta_{abc}=-\frac{\sqrt{2}}{\sqrt{3c_{adj}}}f_{abc}$ gives $\frac{c}{3}=\frac{d_G}{3}$. For $n=m=l$, $\eta_{abc}=\frac{\sqrt{2}}{\sqrt{3n^3}}\sum_{k=0}^{n-1}\exp(2\pi i\frac{k}{n}(a+b+c-3))=\frac{\sqrt{2}}{\sqrt{3n}}\sum_{k=0}^{2}\delta_{a+b+c-3,kn}$ gives $\frac{c}{3}=\frac{n}{3}$. Other solutions are found in [15].

Now $G(z)=-i\eta_{abc}:\bar{\psi}_a\psi_b\psi_c(z):$(where η_{abc} is complex and $\eta_{abc}=\eta_{a[bc]}$) is left to be considered. In general, OPE of GG is nontrivial and the final condition is as complicated as (8), so we shall not write it down in this paper. But for a special form $G(z)=-i\eta_{\alpha\beta a}:\bar{\varphi}_\alpha\varphi_\beta:\psi_a(z)$ ($\eta_{\alpha\beta a}$ is complex), it is easy to show that $\frac{c}{3}$=integer. $G(z)=\frac{1}{\sqrt{2}}(:\bar{\varphi}_1\varphi_1:+i:\bar{\varphi}_2\varphi_2:)\psi(z)$, which can be given by fermionization of $G=i\partial\phi\psi(z)$ (ϕ is a complex boson), is an example of this case.

§4. discussions and conclusions

We have constructed representations of (super)conformal algebras out of fermions but know little about primary fields. Ordinary, fermion $H(z)$ is a primary field with $h=1/2$ and current $J^a(z)=\frac{i}{2}H_\alpha M^a_{\alpha\beta}H_\alpha(z)$ ($M^a_{\alpha\beta}$ is real antisymmetric and satisfy (11)) is a primary field with $h=1$. The precise meaning of them ,however, is that they have $h=1/2$ and 1 with respect to a free energy momentum tensor $T(z)=L(z)=\frac{1}{2}:\partial HH:(z)$ and Sugawara form $T(z)=\mathcal{L}(z)=\frac{1}{\kappa_\lambda+c_{adj}}{}^\circ_\circ J^aJ^a{}^\circ_\circ(z)$ respectively. So in many models we constructed, H and J are not primary fieldes because T is not so. For illustration, we'll consider the discrete series of Virasoro algebra[14]. su(2) current is constructed out of fermions $\psi^0_\alpha,\psi^a_\alpha(a=1,..,N)$ by

$$J^{(1)i}(z)=\frac{1}{2}\bar{\psi}^a_\alpha\sigma^i_{\alpha\beta}\psi^a_\beta(z)\quad(\text{level } N),\qquad J^{(0)i}(z)=\frac{1}{2}\bar{\psi}^0_\alpha\sigma^i_{\alpha\beta}\psi^0_\beta(z)\quad(\text{level } 1),$$

$$J^{(2)i}(z)=J^{(1)i}(z)+J^{(0)i}(z)\quad(\text{level } N+1),$$

and energy momentum tensor $T(z)=K(z)$ is given by

$$K(z)=\mathcal{L}^{(1)}(z)+\mathcal{L}^{(0)}(z)-\mathcal{L}^{(2)}(z)$$

where

$$\mathcal{L}^{(1)}(z)=\frac{1}{N+2}{}^\circ_\circ J^{(1)a}J^{(1)a}{}^\circ_\circ(z),\quad \mathcal{L}^{(0)}(z)=\frac{1}{3}{}^\circ_\circ J^{(0)a}J^{(0)a}{}^\circ_\circ(z),$$

$$\mathcal{L}^{(2)}(z)=\frac{1}{N+3}{}^\circ_\circ J^{(2)a}J^{(2)a}{}^\circ_\circ(z).$$

K(z) has

$$c=1-\frac{6}{(N+2)(N+3)}\,,\qquad h_{pq}=\frac{((N+3)p-(N+2)q)^2-1}{4(N+2)(N+3)}\quad(1\leq q\leq p\leq N+1).$$

In this case ψ,J are no longer primary fields with respect to K(z). But OPE

$$K(z)\psi^a_\alpha(w)\sim\frac{h_{22}}{(z-w)^2}\psi^a_\alpha(w)+\text{less singular term},$$

$$K(z)\psi^0_\alpha(w) \sim \frac{h_{N+1,N+1}}{(z-w)^2}\psi^0_\alpha(w) + \text{less singular term},$$

$$K(z)(J^{(1)i}-NJ^{(0)i})(w) \sim \frac{h_{N+1,N}}{(z-w)^2}(J^{(1)i}-NJ^{(0)i})(w) + \text{less singular term}$$

suggeste that $\psi^a_\alpha+\ldots$, $\psi^0_\alpha+\ldots$ and $J^{(1)i}-NJ^{(0)i}+\ldots$ are primary fields with $h=h_{22}, h_{N+1,N+1}$ and $h_{N+1,N}$.(for N=1 ,i.e. c=1/2, case $\psi^1_\alpha+\ldots$ corresponds to h=1/16 and $J^{(1)}-J^{(0)}$ corresponds to h=1/2). Here "+..." is very complicated infinite series. We hope it has a concise form like as a vertex operator.

In this talk we discussed fermionic construction of N=0,1 and 2 (super)conformal algebra. Starting from general form of supercurrents, we derived conditions for them to realize conformal algebra. In particular, it was shown that, in N=2 case, these conditions reduced to a simpler form when we use complex fermions. We gave several solutions which satisfy these conditions. Our construction will give a step forward in understanding and studying the structure of superconformal algebras.

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