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Continuum Limit of Spin-1 Chain

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Abstract

We study the continuum limit of the spin-1 chain in the non-Abelian bosonization approach of Affleck and show that the Hamiltonian of integrable spin-1 chain yields the Lagrangian of supersymmetric sine-Gordon model in the zero lattice spacing limit. We also show that the quantum group generators of the spin-1 chain give non-local charges of the supersymmetric sine-Gordon theory.

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One-dimensional quantum spin chains have played a very important role in the theory of highly correlated electron systems. There has recently been experimental interest in quasi one-dimensional systems of loosely coupled molecules. They can be described (approximately) by anti-ferromagnetic spin chains. Some of them have spin s greater than $\frac{1}{2}$. Spin chains have also played a special role in the theory of integrable systems. XXZ spin chains are known to have quantum group symmetry $U_q su(2)$ [1]. This symmetry underlies the integrability of spin chains. Quantum group symmetries[2] also appear in some class of quantum field theories in $1+1$ dimensions, e.g. non-local conserved charges in the sine-Gordon theory generate the affine $U_q\widehat{su}(2)$ symmetry[3]. In fact, the continuum (field theory) limit of the XXZ spin-$\frac{1}{2}$ chain is known to be the sine-Gordon theory[4, 5].

In this letter we consider the continuum limit of the spin-1 chain in the non-Abelian bosonization approach of Affleck[6]. We show that the Hamiltonian of the integrable spin-1 chain yields the Lagrangian of supersymmetric sine-Gordon model in the zero lattice spacing limit and that the $U_q\widehat{su}(2)$ quantum group generators of the infinitely long chain give non-local charges of the latter theory.

It is believed that, in contrast to the spin-$\frac{1}{2}$ case, the spin-1 Heisenberg model has a mass gap[7]. The more general spin-1 chain with isotropic bilinear-biquadratic Hamiltonian $H = J\sum_n[\vec{S}_{n+1}\cdot\vec{S}_n - b(\vec{S}_{n+1}\cdot\vec{S}_n)^2]$ is also conjectured to be gapful for $b \neq \pm 1$. We are interested in the case of $b = 1$, which is an integrable point and gapless[8], and hence it makes sense to consider its field theory limit. Integrable deformation of this Hamiltonian incorporating anisotropy was constructed in ref.[9] and was further studied in ref.[10].

$$\begin{aligned} H_{XXZ} \;=\; & J\sum_n\Big[\vec{S}_{n+1}\cdot\vec{S}_n - (\vec{S}_{n+1}\cdot\vec{S}_n)^2 \\ & +\tfrac{1}{2}(q-q^{-1})^2(S^3_{n+1}S^3_n - (S^3_{n+1}S^3_n)^2 + (S^3_{n+1})^2 + (S^3_n)^2) \\ & -\tfrac{1}{2}(q+q^{-1}-2)\{S^3_{n+1}S^3_n, S^+_{n+1}S^-_n + S^-_{n+1}S^+_n\} + \tfrac{1}{2}(q^2-q^{-2})(S^3_{n+1}-S^3_n)\Big]. \end{aligned} \tag{1}$$

This XXZ Hamiltonian commutes with $U_q su(2)$, whose generators are H_1 and $E^\pm_1$:

$$q^{H_1} = \cdots q^{2S^3_{n+1}}q^{2S^3_n}q^{2S^3_{n-1}}\cdots, \quad E^\pm_1 = \sqrt{[2]/2}\sum_n \cdots q^{S^3_{n+1}}S^\pm_n q^{-S^3_{n-1}}\cdots, \tag{2}$$

where $S^\pm = S^1 \pm iS^2$ and $[x] = (q^x - q^{-x})/(q-q^{-1})$. For infinitely long chain, in which case the last boundary term can be discarded, this symmetry is enhanced to the affine quantum group symmetry $U_q\widehat{su}(2)$ with level 0 [11], whose generators are H_1, $E^\pm_1$, $H_0 = -H_1$ and $E^\pm_0 = \sqrt{[2]/2}\sum_n \cdots q^{-S^3_{n+1}}S^\mp_n q^{S^3_{n-1}}\cdots$.

To derive the continuum limit of (1) we use the oscillator representation of spin-1 operator S_n^a. In view of the lack of the Jordan-Wigner transformation for the spin-1 case, we employ the construction of S_n^a out of a pair of spin-$\frac{1}{2}$ operators[6, 5]. We will comment on other methods later. To get $s=1$ we need two ($f=1,2$) replicas of doublet ($\alpha=1,2$) complex fermions $\psi_n^{\alpha f}$. They obey the canonical anticommutation relations $\{\psi_n^{\alpha f}, \psi_m^{\beta g \dagger}\} = \delta^{\alpha\beta}\delta^{fg}\delta_{nm}$. The spin-1 operator is represented as

$$S_n^a = \tfrac{1}{2}\psi_n^{\alpha f\dagger}\sigma_{\alpha\beta}^a\psi_n^{\beta f}, \tag{3}$$

where σ^a's are Pauli matrices. On each lattice site there are 2^4 states obtained by acting creation operators $\psi_n^{\alpha f\dagger}$ on the vacuum $|0\rangle_n$ defined by $\psi_n^{\alpha f}|0\rangle_n = 0$. Spin-1 states $|1,m\rangle_n$ are obtained by acting the lowering operator S_n^- on the highest weight state $|1,1\rangle_n = \prod_f \psi_n^{1f\dagger}|0\rangle_n$. This spin-1 representation space is characterized by[6]

$$\vec{S}_n^2|*\rangle^L = 2|*\rangle^L, \quad \text{or equivalently} \quad \psi_n^{\alpha f\dagger}\psi_n^{\alpha g}|*\rangle^L = \delta^{fg}|*\rangle^L. \tag{4}$$

Here the suffix L refers to the lattice theory. Thus we impose the above constraint to project out three spin-1 states $|1,m\rangle_n$ from 16 states on each site.

The second equation of (4) means that a half of the particle states is filled on each site. In the case of spin-$\frac{1}{2}$, an analysis of the Hubbard model shows that this condition implies half-filling in the momentum space. By a similar analysis of multi-band Hubbard models, one can argue that this is the case for higher spins[6, 5]. We define the half-filling vacuum $|0\rangle^{HF}$ as the state in which particle states are filled up to the Fermi sea of momentum $k_F = \pi/2a$. Low energy excitations are creations of fermions and holes near the Fermi sea. To describe such excitations we introduce chiral fermions $\psi_{\pm,\ell}^{\alpha f}$ on a pair of even and odd lattice sites:

$$\psi_{2\ell}^{\alpha f} = (-1)^\ell(\psi_{+,\ell}^{\alpha f} + \psi_{-,\ell}^{\alpha f})/\sqrt{2}, \quad \psi_{2\ell-1}^{\alpha f} = -i(-1)^\ell(\psi_{+,\ell}^{\alpha f} - \psi_{-,\ell}^{\alpha f})/\sqrt{2}. \tag{5}$$

The fast oscillating term $(-1)^\ell$ comes from $\exp(\pm ik_F a2\ell)$ and $\exp(\pm ik_F a(2\ell-1))$. The spin operator (3) is expressed in terms of chiral fermions as

$$S_{2\ell}^a = \tfrac{1}{2}(J_\ell^a + G_\ell^a), \quad S_{2\ell-1}^a = \tfrac{1}{2}(J_\ell^a - G_\ell^a), \tag{6}$$

where

$$J_{\pm,\ell}^a = \tfrac{1}{2}\psi_{\pm,\ell}^{\alpha f\dagger}\sigma_{\alpha\beta}^a\psi_{\pm,\ell}^{\beta f}, \quad J_\ell^a = J_{+,\ell}^a + J_{-,\ell}^a, \quad G_\ell^a = \tfrac{1}{2}(\psi_{+,\ell}^{\alpha f\dagger}\sigma_{\alpha\beta}^a\psi_{-,\ell}^{\beta f} + \psi_{-,\ell}^{\alpha f\dagger}\sigma_{\alpha\beta}^a\psi_{+,\ell}^{\beta f}). \tag{7}$$

We will present the derivation of the continuum limit of the Hamiltonian H of isotropic spin-1 chain. Extension to the anisotropic case H_{XXZ} will be briefly discussed later. We take the zero lattice spacing limit ($a \to 0$). The space coordinate is $x = 2a\ell$ and the sum is replaced by the integral $2a\sum_\ell \to \int dx$. There are a few alternative ways of computing the continuum Hamiltonian depending on the different stages at which we move from the lattice to continuum theory. We take the prescription of taking the $a \to 0$ limit in an early stage and computing the operator products of currents in the continuum theory. We have also made the computation in the lattice theory taking the $a \to 0$ limit in the resulting expression. We have obtained the same physical results (modulo some subtleties related to regularization).

In the continuum $\frac{1}{\sqrt{2a}}\psi^{\alpha f}_{\pm,\ell} \to \psi^{\alpha f}_{\pm}(x)$ and their propagators are $\langle \psi^{\alpha f \dagger}_{\pm}(x)\psi^{\beta g}_{\pm}(y)\rangle = \delta^{\alpha\beta}\delta^{fg}(\mp 2\pi i)^{-1}(x - y \pm i\epsilon)^{-1}$, where $\pm i\epsilon$ is the UV cutoff in the continuum theory. We assume the existence of the continuum (field theory) limit of lattice spin-1 states and the half-filled vacuum: $|*\rangle^{L} \to |*\rangle^{FT}$, $|0\rangle^{HF} \to |0\rangle^{FT}$. Here suffices L, FT and HF refer to lattice, field theory and half filling. Normal ordering of fermions refers to this $|0\rangle^{FT}$. Currents in the continuum are

$$(2a)^{-1}J^a_{\pm,\ell} \;\to\; J^a_{\pm}(x) = \tfrac{1}{2}(\psi^{\alpha f\dagger}_{\pm}\sigma^a_{\alpha\beta}\psi^{\beta f}_{\pm})(x), \tag{8}$$

$$(2a)^{-1}G^a_{\ell} \;\to\; G^a(x) = \tfrac{1}{2}(\psi^{\alpha f\dagger}_{+}\sigma^a_{\alpha\beta}\psi^{\beta f}_{-} + \psi^{\alpha f\dagger}_{-}\sigma^a_{\alpha\beta}\psi^{\beta f}_{+})(x) \tag{9}$$

and their operator product expansions (OPE) are easily calculated. For example,

$$J^a_{\pm}(x)J^b_{\pm}(0) \;=\; (\mp 2\pi i x)^{-2}\delta^{ab} + (\mp 2\pi i x)^{-1} i\epsilon^{abc}J^c_{\pm}(0) + (J^a_{\pm}J^b_{\pm})(0) + \cdots, \tag{10}$$

$$J^a_{\pm}(x)G^b(0) \;=\; (\mp 2\pi i x)^{-1}(\pm\tfrac{1}{4}\delta^{ab}F(0) + \tfrac{1}{2}i\epsilon^{abc}G^c_{\pm}(0)) + (J^a_{\pm}G^b)(0) + \cdots, \tag{11}$$

where $F(x) = (\psi^{\alpha f\dagger}_{+}\psi^{\alpha f}_{-} - \psi^{\alpha f\dagger}_{-}\psi^{\alpha f}_{+})(x)$. The first equation means that $J^a_{\pm}$ define the $\widehat{su}(2) \times \widehat{su}(2)$ Kac-Moody algebra of level $k = 2$, as we have designed. Following Affleck we assume that the states in the field theory satisfy the continuum limit of the spin-1 constraint (4)[6]

$$(\vec{J}^{\,2} + \vec{G}^2)(x)|*\rangle^{FT} = 0, \quad (\vec{J}\cdot\vec{G} + \vec{G}\cdot\vec{J})(x)|*\rangle^{FT} = 0, \tag{12}$$

where $(AB)(x)$ stands for normal ordering defined by the regular part of OPE.

We are now ready to compute the $a \to 0$ limit of the Hamiltonian H. Using $(\vec{S}_{n+1}\cdot\vec{S}_n)^2 = \frac{1}{4}\{S^a_{n+1}, S^b_{n+1}\}\{S^a_n, S^b_n\} - \frac{1}{2}\vec{S}_{n+1}\cdot\vec{S}_n$, the Hamiltonian is now written as

$$H = 2aJ\int dx\Big[\tfrac{3}{2}(\mathcal{H}^{(2)}_e + \mathcal{H}^{(2)}_o)(x) - (\mathcal{H}^{(4)}_e + \mathcal{H}^{(4)}_o)(x)\Big], \tag{13}$$

where suffices e and o refer to even and odd n, and

$$(2a)^{-2}\vec{S}_{2\ell+1}\cdot\vec{S}_{2\ell} \quad\rightarrow\quad \mathcal{H}_e^{(2)}(x)=\tfrac{1}{4}(\vec{J}-\vec{G})(x+2a)\cdot(\vec{J}+\vec{G})(x), \tag{14}$$

$$(2a)^{-2}\tfrac{1}{4}\{S^a_{2\ell+1},S^b_{2\ell+1}\}\{S^a_{2\ell},S^b_{2\ell}\} \quad\rightarrow$$

$$\mathcal{H}_e^{(4)}(x)=(2a)^2\tfrac{1}{16}[(\pi\epsilon)^{-2}\delta^{ab}+(J^aJ^b+G^aG^b)(x+2a)-(J^aG^b+G^aJ^b)(x+2a)]$$
$$\times[(\pi\epsilon)^{-2}\delta^{ab}+(J^aJ^b+G^aG^b)(x)+(J^aG^b+G^aJ^b)(x)], \tag{15}$$

and similar expressions for $\mathcal{H}_o^{(2)}$ and $\mathcal{H}_o^{(4)}$. We have used the fact that $(J^aJ^b+G^aG^b)$ and $(J^aG^b+G^aJ^b)$ are symmetric in a and b. The relevant terms of the Hamiltonian can be obtained using the OPE such as (10) and (11). The results are

$$\mathcal{H}_e^{(2)}+\mathcal{H}_o^{(2)} \;=\; \tfrac{1}{2}(\vec{J}^{\,2}-\vec{G}^2), \tag{16}$$

$$\mathcal{H}_e^{(4)}+\mathcal{H}_o^{(4)} \;=\; -1/(2\pi)^2[\tfrac{1}{2}(\vec{J}^{\,2}-\vec{G}^2)+\tfrac{1}{2}(\vec{K}^2)+\tfrac{3}{4}(F^2)]$$
$$+(a/\pi\epsilon)^2\tfrac{1}{2}(\vec{J}^{\,2}+\vec{G}^2)-(2a)^{-1}15i/(32\pi^3)F, \tag{17}$$

where $\vec{K}=\vec{J}_+-\vec{J}_-$ (the coefficients depend on how the continuum theory is regularized). There appear operators F, $\vec{G}^2$ and F^2 in addition to the composites of the currents $J^a_\pm$ of the $\widehat{su}(2)$ Kac-Moody algebra of level 2, which we denote by $\widehat{su}(2)_2$. The divergent term $a^{-1}F$ violates the invariance under the translation by a. We should discard this term assuming the lattice regularization respecting this invariance.

Introducing time t, the Hamiltonian can be converted to the Lagrangian

$$\mathcal{L}=\tfrac{1}{2}i(\psi_+^{\alpha f\dagger}\overleftrightarrow{\partial_0}\,\psi_+^{\alpha f}+\psi_-^{\alpha f\dagger}\overleftrightarrow{\partial_0}\,\psi_-^{\alpha f})-\mathcal{H}. \tag{18}$$

As we have designed, $\psi_\pm^{\alpha f}(x)$ become right(left)-moving fermions $\psi_\pm^{\alpha f}(x^\pm)$, where $x^\pm=x^0\mp x^1=t\mp x$. By evaluating the operator products of $G^a(x^+,x^-)$ and $F(x^+,x^-)$ with $J^a_\pm(x^\pm)$, we have found that G^a and F are spin $(\frac{1}{2},\frac{1}{2})$ multiplet of $\widehat{su}(2)_2\times\widehat{su}(2)_2$ and $(\vec{G}^2-\frac{1}{4}F^2)$ is a singlet. This implies that we should set $(F^2-4\vec{G}^2)(x)=0$. After using this relation and the constraint (12), the Hamiltonian is

$$H \;=\; 2aJ\int dx[A(\vec{J}_+^{\,2}+\vec{J}_-^{\,2})(x)+2B(\vec{J}_+\cdot\vec{J}_-)(x)], \tag{19}$$

$$A \;=\; 3[1-1/(2\pi)^2]/2,\quad B=[3-5/(2\pi)^2]/2. \tag{20}$$

We now begin to see the emergence of the supersymmetric sine-Gordon theory. After normalizing J correctly($J\sim a^{-1}$), the first term on the r.h.s. of (19) is the Hamiltonian of the Wess-Zumino-Witten model with level 2. This model has the central charge $c = 3/2$ and is supersymmetric. The second term is a perturbation to the conformal invariant theory and the resulting theory is the super sine-Gordon theory with $\beta = \sqrt{4\pi}$ [12, 13].

To express the Hamiltonian in a more familiar form, we use the fact that the Kac-Moody algebra $\widehat{su}(2)_2$ is represented by a real boson and a real fermion ($\mathbf{Z}_2$ parafermion)[9]:

$$J^3_\pm(x^\pm) = \pm\sqrt{1/\pi}\partial_\pm\phi_\pm(x^\pm), \quad J^+_\pm(x^\pm) = \psi_\pm(x^\pm)\sqrt{\mu/\pi}e^{\pm i\sqrt{4\pi}\phi_\pm(x^\pm)}, \quad J^-_\pm = (J^+_\pm)^\dagger, \tag{21}$$

where we have suppressed the normal ordered symbol and "cocycle factors" which ensure the commutativity of right and left currents. The propagators of $\phi_\pm$ and $\psi_\pm$ are $\langle\phi_\pm(x^\pm)\phi_\pm(0)\rangle = -\frac{1}{4\pi}\log(i\mu x^\pm)$, $\langle\psi_\pm(x^\pm)\psi_\pm(0)\rangle = (2\pi i x^\pm)^{-1}$. Using the identity[14] $\mu^2\cos^2(\sqrt{4\pi}\phi) = \frac{\pi}{2}(\partial\phi)^2$, the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{i}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{1}{4}\lambda\beta\bar{\psi}\psi\cos(\beta\phi) + \frac{1}{8}\lambda^2\beta^2\cos^2(\beta\phi), \tag{22}$$

where $\beta = \sqrt{4\pi}$ and $\lambda = \mu B/2\pi A$. Here $\psi = (\psi_+, \psi_-)^T$. This is the super sine-Gordon model with $\beta = \sqrt{4\pi}$. For this value of β the last two terms in (22) are (irrelevantly) marginal.

To get the model with $\beta < \sqrt{4\pi}$ we should consider the anisotropic case H_{XXZ}. A new feature is that the Hamiltonian (1) contains the (G^3G^3) term. In the case of spin-$\frac{1}{2}$, the (G^3G^3) term can be expressed in terms of $\widehat{su}(2)_1$ currents $J^a_\pm$ after solving the constraint like (4). In the case of spin-1, we expect that the (G^3G^3) term can be expressed in terms of $\widehat{su}(2)_2$ currents $J^a_\pm$ by solving the constraint (4). Then, the anisotropic terms (J^3J^3) result in a change of the normalization of ϕ and we must rescale $\phi \to \frac{\beta}{\sqrt{4\pi}}\phi$, as discussed in ref.[5] in the case of spin-$\frac{1}{2}$. Finally we get the Lagrangian of supersymmetric sine-Gordon theory (22) with a general value of β. Unfortunately we are not yet able to solve the constraint (4) explicitly.

Next we consider the continuum limit of the $U_q\widehat{su}(2)_0$ generators (2). They are rewritten, without any approximation, as $q^{H_1} = q^{2\sum_\ell J^3_\ell}$ and

$$E^\pm_1 = \sqrt{[2]/2}\sum_\ell q^{\sum_{\ell'>\ell}J^3_{\ell'}}\left[S^\pm_{2\ell}q^{-S^3_{2\ell-1}} + q^{S^3_{2\ell}}S^\pm_{2\ell-1}\right]q^{-\sum_{\ell'<\ell}J^3_{\ell'}}. \tag{23}$$

Taking the continuum limit, they are expressed in terms of $J^a_\pm$ as $q^{H_1} = q^{2\int dxJ^3(x)}$ and

$$E^\pm_1 = \sqrt{[2]/2}\int_{-\infty}^{\infty}dxq^{\int_x^\infty dx'J^3(x')}J^\pm(x)q^{-\int_{-\infty}^x dx'J^3(x')}, \tag{24}$$

and similar expressions for H_0 and $E_0^{\pm}$. The chiral bosons $\phi_\pm$ are represented in terms of ϕ and its conjugate momentum $\pi = \partial_0\phi$ as $\phi_\pm(t,x) = \frac{1}{2}[\phi(t,x) \mp \int_{-\infty}^{x} dx'\pi(t,x')]$. Under the rescaling $\phi \to \frac{\beta}{\sqrt{4\pi}}\phi$, the conjugate momentum must rescale as $\pi \to \frac{\sqrt{4\pi}}{\beta}\pi$. After expressing $J_\pm^a$ in terms of $\phi_\pm$ and $\psi_\pm$, and rescaling ϕ, quantum group generators becomes $q^{H_1} = q^{-\frac{\beta}{\pi}(\phi(\infty)-\phi(-\infty))}$ and

$$E_1^{\pm} = \sqrt{[2]/2}\sqrt{\mu/\pi}q^{-\frac{\beta}{2\pi}(\phi(\infty)+\phi(-\infty))}\int_{-\infty}^{\infty} dx(\psi_+ e^{i\frac{4\pi}{\beta}\phi_+} + \psi_- e^{i\frac{4\pi}{\beta}\phi_+ - i\beta\phi}), \tag{25}$$

and similar expressions for other generators, where $q = e^{i2\pi^2/\beta^2 - i\pi/2}$. These expressions agree with the non-local charges in the supersymmetric sine-Gordon theory[13] up to some constant factor. We can also show that the continuum limit of the quantum group generators of the spin-$\frac{1}{2}$ XXZ chain agree with the non-local charges in the sine-Gordon theory[3].

We comment on other oscillator representations of the spin-1 operator:
(i) Spin-1 version of Jordan-Wigner transformation. Jordan-Wigner transformation for spin-$\frac{1}{2}$ case has an advantage that there are no constraints like (4). This is because the Fock space on each site ($|0\rangle_n$, $\psi^\dagger|0\rangle_n$, ($\psi^{\dagger 2}|0\rangle_n = 0$)) agrees with the spin-$\frac{1}{2}$ representation space. The spin-1 version is to introduce a "parafermion" such that its Fock space on each site is three dimensional ($|0\rangle_n$, $\psi^\dagger|0\rangle_n$, $\psi^{\dagger 2}|0\rangle_n$, ($\psi^{\dagger 3}|0\rangle_n = 0$)), which can be identified with the spin-1 representation space.
(ii) Triplet real fermions. The supersymmetric sine-Gordon Hamiltonian is expressed in terms of the $\widehat{su}(2)_2$ currents, and the level 2 currents are realized by a triplet of real fermions[9]. It seems natural to introduce triplet real fermions from the beginning and write $S_n^a = -\frac{1}{2}i\epsilon^{abc}\psi_n^b\psi_n^c$. Real fermions on each site, however, do not allow a definite particle picture. S_n^a acts on ψ_n^a as spin-1 representation by adjoint action $[S_n^a,[S_n^a,\psi_n^b]] = 2\psi_n^b$, and $S_n^a S_n^a = \frac{3}{4} \neq 2$, in contrast with (4). Nevertheless it is tempting to pursue this possibility further. In this construction of S_n^a there appear the operators F and G^a which obey the OPE similar to those discussed above. The G^a can be shown to satisfy the constraint (12). The computation of the continuum Hamiltonian is straightforward and we get the same form as (19) and hence the supersymmetric sine-Gordon theory. Presumably the field theory treatment of the spin-1 chain suggested by Tsvelik[15] can be derived in this way.

Our derivation of the supersymmetric sine-Gordon theory as the continuum limit of XXZ spin-1 chain is rather heuristic. However, the fact that the connection of the quantum group generators in the lattice theory and those in the continuum theory is correctly obtained supports our conclusion. A rigorous proof can be made by carrying out an analysis based on the Bethe ansatz similar to that used to prove the equivalence of the continuum limit of XXZ(XYZ) spin-$\frac{1}{2}$ chain and the sine-Gordon theory[4].

We have shown that the continuum theory possesses supersymmetry. The question arises whether the spin-1 chain has supersymmetry for finite lattice spacing or supersymmetry emerges only in the zero lattice spacing. This question can be answered by making a more rigorous treatment mentioned above.

The present approach of deriving the continuum limit can be applied to other cases of integrable spin chains: (a) The spin-1 Hamiltonian with $b = -1$ is known to have $SU(3)$ symmetry[16]. We expect to get the affine $\widehat{su}(3)$ Toda field theory in the continuum limit. (b) For the higher spin case we introduce $2s$ doublets of fermions to express the spin operator. We expect to get the fractional supersymmetric sine-Gordon theory[17] in the continuum.

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